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Local Measurement Scheme of Gravitational Curvature using Atom Interferometers

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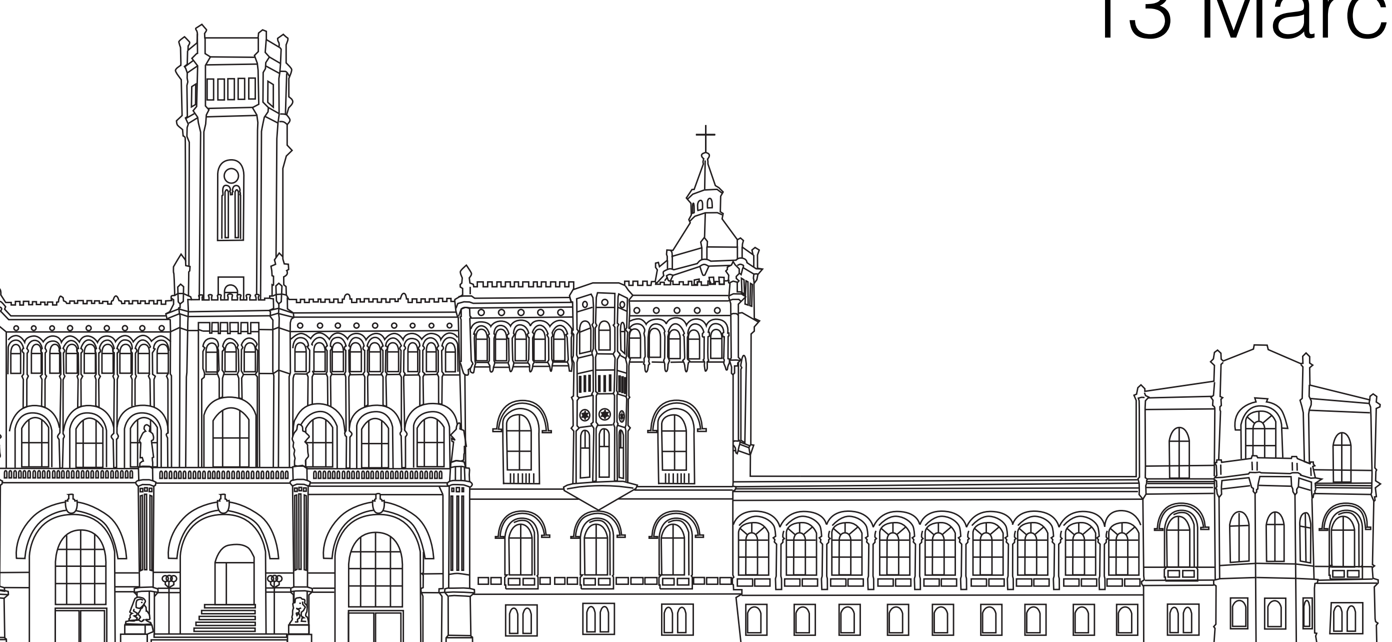
13 March 2025



Q 53: Matter Wave Interferometry II
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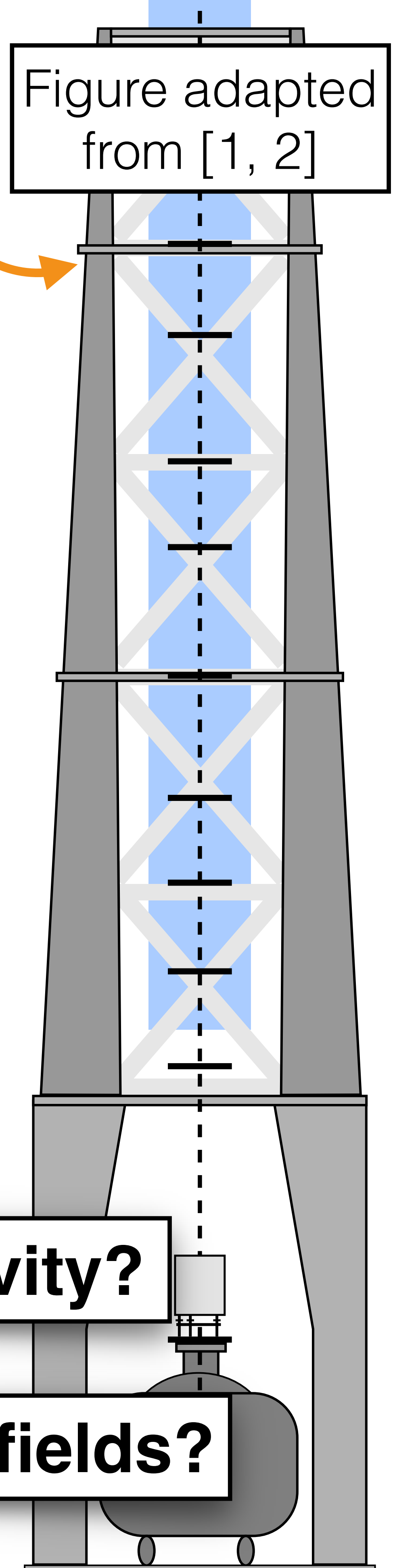
Motivation



Atom interferometers like the VLBAI in Hannover are highly accurate sensors for the gravitational field.

If you consider a gravitational potential like

$$\phi(z) = \phi_0 + gz - \frac{1}{2}\Gamma_0 z^2$$

One can mitigate [3, 4] the effect of Γ_0 and accurately measure the value of g.

Question 1: Is this the only way to measure gravity?

Question 2: How does this work in complex grav. fields?

Contents

Present our novel paper

Local Measurement Scheme of Gravitational Curvature using Atom Interferometers

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(Dated: October 7, 2024)

Light pulse atom interferometers (AIFs) are exquisite quantum probes of spatial inhomogeneity and gravitational curvature. Moreover, detailed measurement and calibration are necessary prerequisites for very-long-baseline atom interferometry (VLBAI). Here we present a method in which the differential signal of two co-located interferometers singles out a phase shift proportional to the curvature of the gravitational potential. The scale factor depends only on well controlled quantities, namely the photon wave number, the interferometer time and the atomic recoil, which allows the curvature to be accurately inferred from a measured phase. As a case study, we numerically simulate such a co-located gradiometric interferometer in the context of the Hannover VLBAI facility and prove the robustness of the phase shift in gravitational fields with complex spatial dependence. We define an estimator of the gravitational curvature for non-trivial gravitational fields and calculate the trade-off between signal strength and estimation accuracy with regard to spatial resolution. As a perspective, we discuss the case of a time-dependent gravitational field and corresponding measurement strategies.

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I. INTRODUCTION

AIFs are high-precision instruments used in a wide variety of research fields. Their versatility includes tasks such as determining the fundamental constants [1–4], serving as quantum sensors to measure Earth's gravitational field [5–7], proposing measurements for gravitational wave detection [8–10], exploring fundamental physics and alternative gravitational models [11–14], and performing measurements related to time dilation and gravitational redshift [15–18]. In particular, their accuracy as sensors of gravitational fields and their gradients is becoming increasingly important for applications in civil engineering [19], inertial sensing [20] and geodesy [21–26].

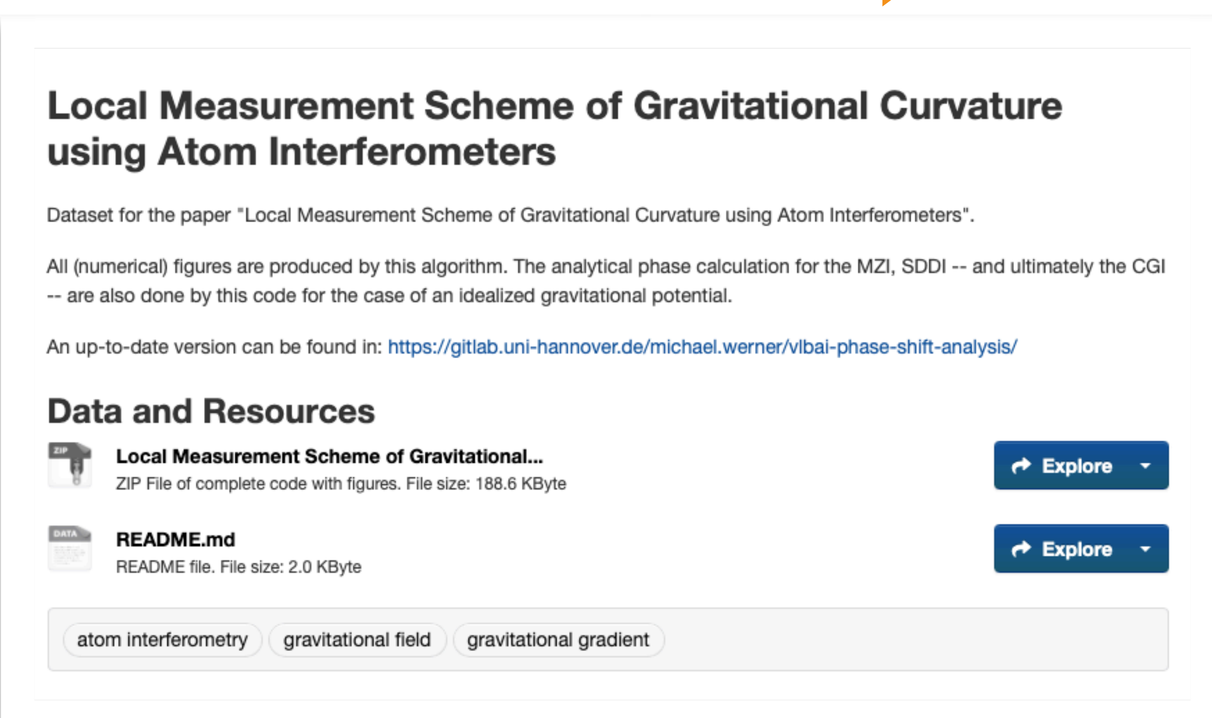
AIFs are utilized to measure the gravitational field, there they provide information about the linear gravitational acceleration g along the atomic trajectory. This approach is highly accurate because the leading order phase shift $\Delta\Phi = gkT_R^2$ connects the desired value of g with the wave vector k and the interferometer time T_R, both of which are known with very high precision. For measuring the (constant) gravitational gradient, a gradiometric experimental setup is employed, involving a comparison of g-measurements from two spatially separated gravimeters, effectively interpolating the g values between their spatial positions. Such gradiometric experiments are theoretically limited by the measurement uncertainty of the phase shift and the uncertainty of the height difference between the two interferometers. Another way to extract knowledge about the gravity gradient is done using more elaborate AIF geometries [27]. In these cases, however, the phase shift depends non-linearly on the gravitational field, making an estimation more complicated.

State-of-the-art AIFs are being constructed with increasingly longer baselines [28–31] and more efficient large momentum transfer (LMT) techniques [32–34], extending beyond the region where the assumption of a constant gradient of the gravitational field remains valid. The transition to non-trivial gravitational curvature is not only a challenge for large baseline interferometers, but can also be seen as an opportunity for experiments with gravitational test masses. Deliberately introduced non-trivial gravitational fields, which allow the measurement of phases along the atomic trajectory to probe this non-linearity, have been exploited in [35, 36] and led to the proposed gravitational Aharonov-Bohm effect [37]. Measuring anomalies in the gravitational gradient is also used to detect inhomogeneities in the gravitational field [19] and will become evermore important for civil engineering and quantum metrology. Resolving a spatially varying gravity gradient to high accuracy with a gradiometric AIF setup is, however, equivalent to comparing g-measurements in close proximity. This procedure is therefore increasingly error prone, because of the relative uncertainty in the position of the atomic ensembles, compared to the separation of the two constituent AIFs.

In this analysis, we introduce a novel geometry for AIFs that is exclusively sensitive to the gravitational curvature, that is,

FIG. 1. Depiction of the co-located gradiometric interferometer (CGI) setup consisting of a SDDI (green) and a MZI (blue) in a gravitational field sourced by the mass density $\rho(\boldsymbol{r}, t)$. (a) Position of the CGI in a large baseline interferometry setup as as determined by the initial height z_0. CGI geometry shown in more detail (b) in the laboratory frame and (c) in the freely falling frame. $|N\rangle$ denotes a momentum eigenstate with N momentum quanta, as compared to the initial wave packet. The speed of light was set infinite for the laser pulses in this plot.

Open source Python algorithm (generates all following figures)



Local Measurement Scheme of Gravitational Curvature using Atom Interferometers

Dataset for the paper "Local Measurement Scheme of Gravitational Curvature using Atom Interferometers".

All (numerical) figures are produced by this algorithm. The analytical phase calculation for the MZI, SDDI -- and ultimately the CGI -- are also done by this code for the case of an idealized gravitational potential.

An up-to-date version can be found in: https://gitlab.uni-hannover.de/michael.werner/vlbai-phase-shift-analysis/

Data and Resources

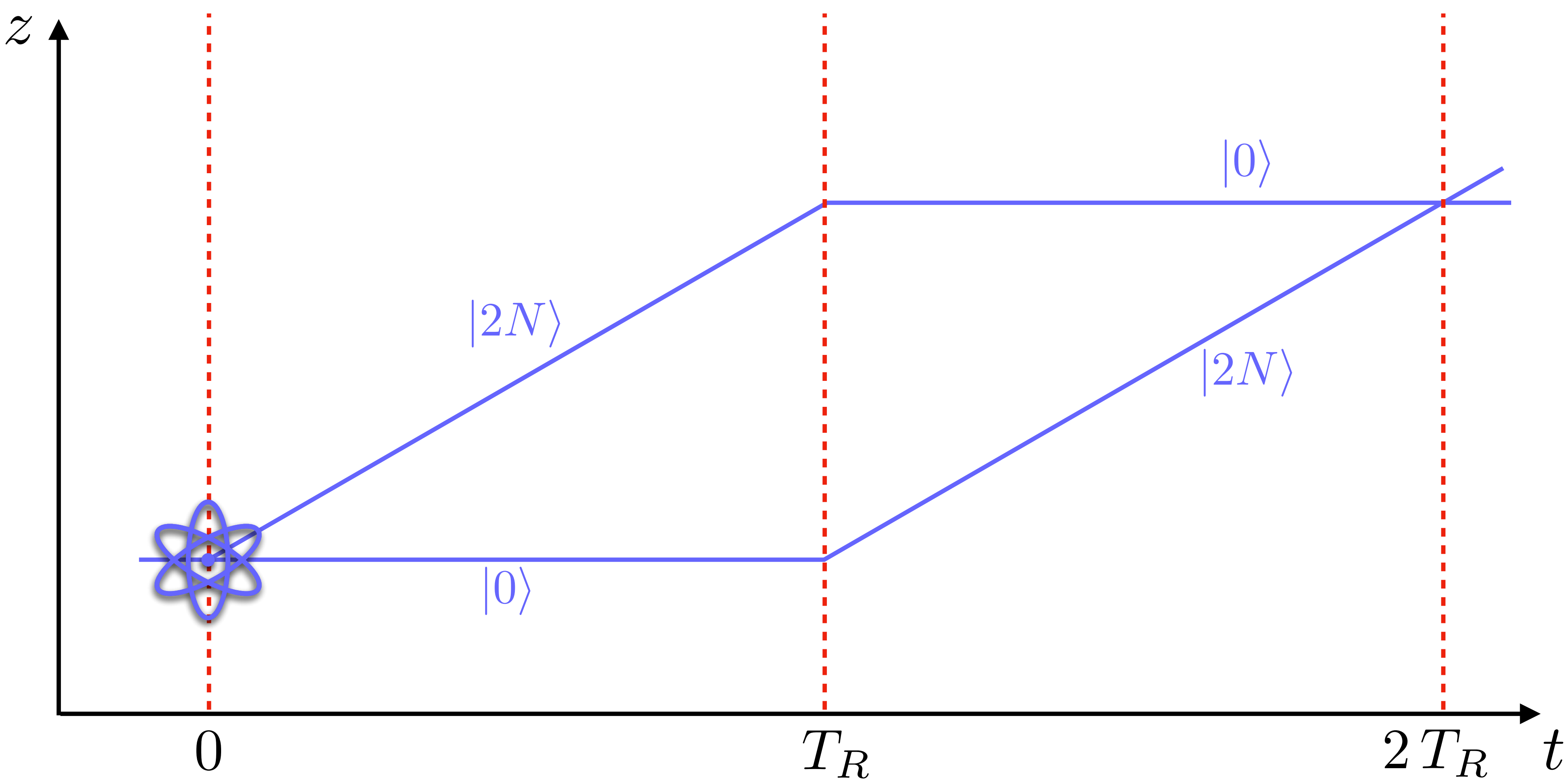
Local Measurement Scheme of Gravitational...
ZIP File of complete code with figures. File size: 188.6 KByte
Explore

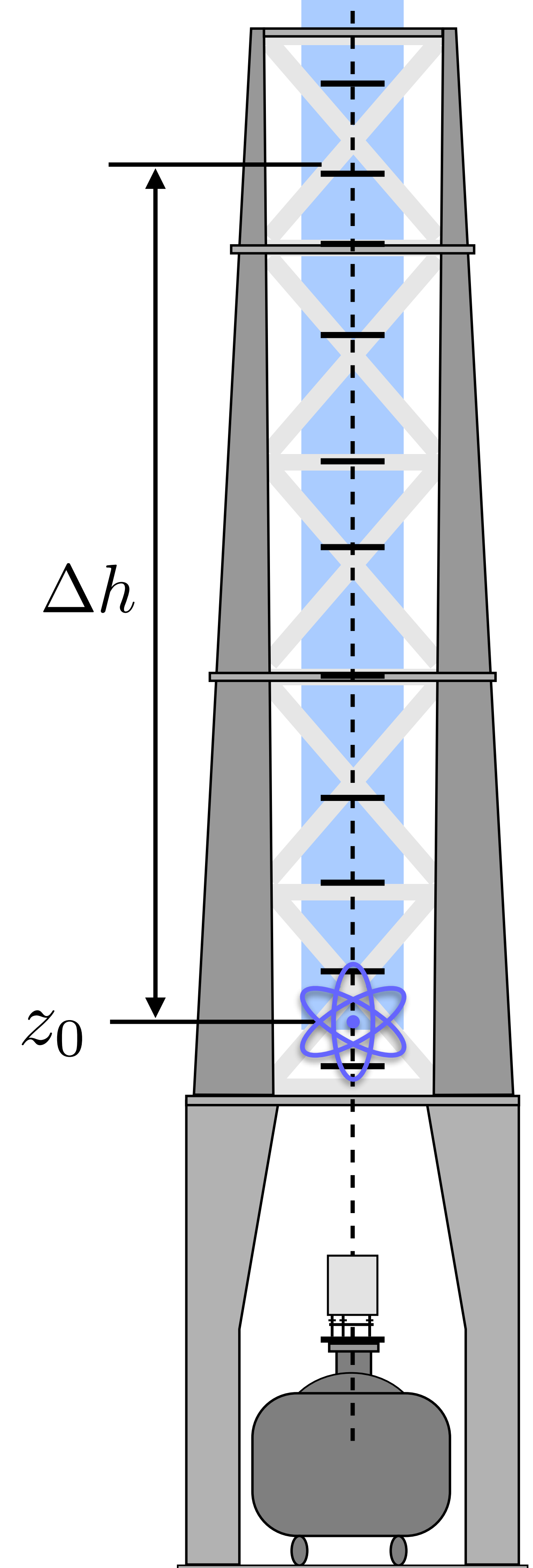
README.md
README file. File size: 2.0 KByte
Explore

atom interferometry | gravitational field | gravitational gradient

Gravimetry in a nutshell

Using a Mach-Zehnder interferometer (MZI)





Phase shift:

$$\Delta\Phi = 2NgkT_R^2$$

Estimator for gravity:

$$\hat{g} = \frac{\Delta\Phi}{2NkT_R^2}$$

Known to high accuracy

From Gravimetry to Gradiometry

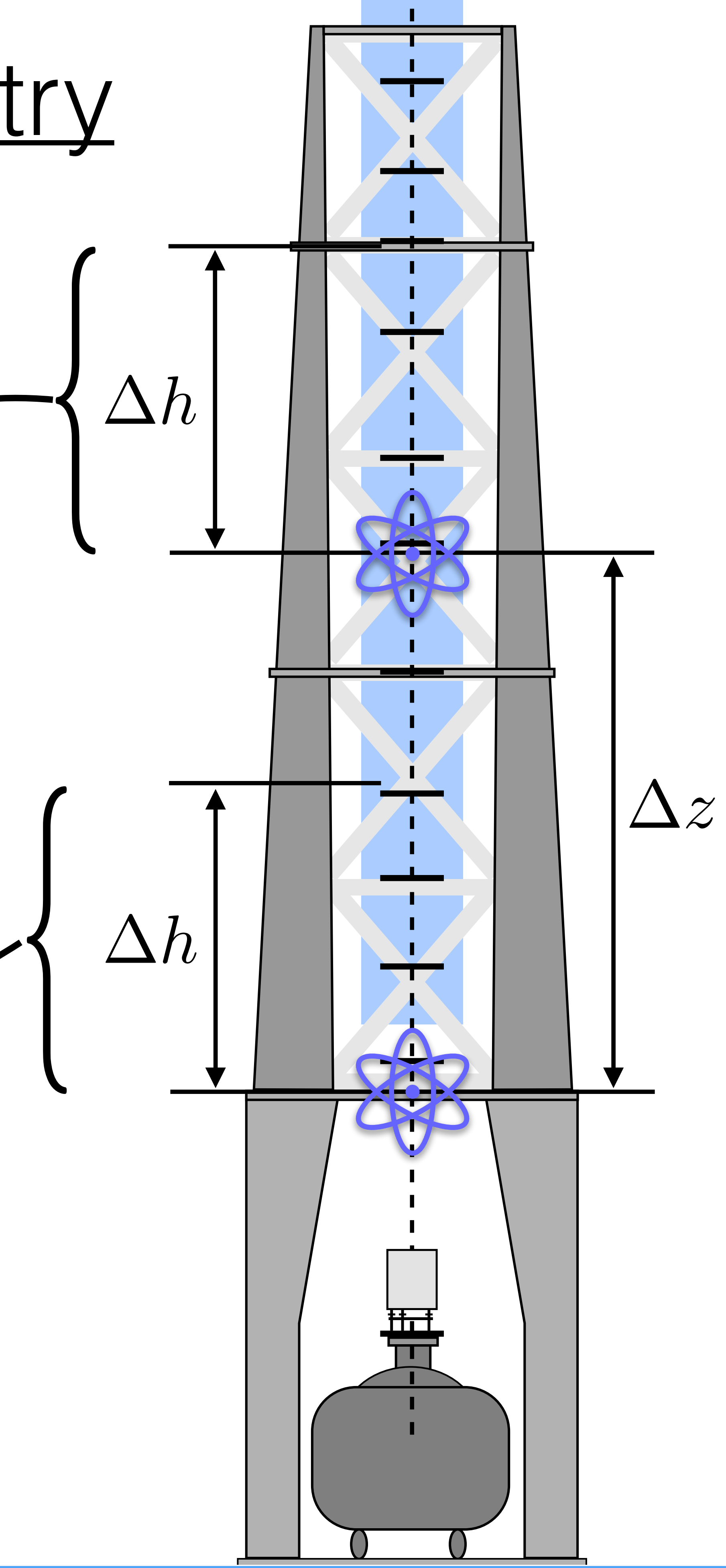
Usual procedure for gradiometry:

$$g(z_0 + \Delta z)$$

$$\Delta g = g(z_0 + \Delta z) - g(z_0)$$

$$g(z_0)$$

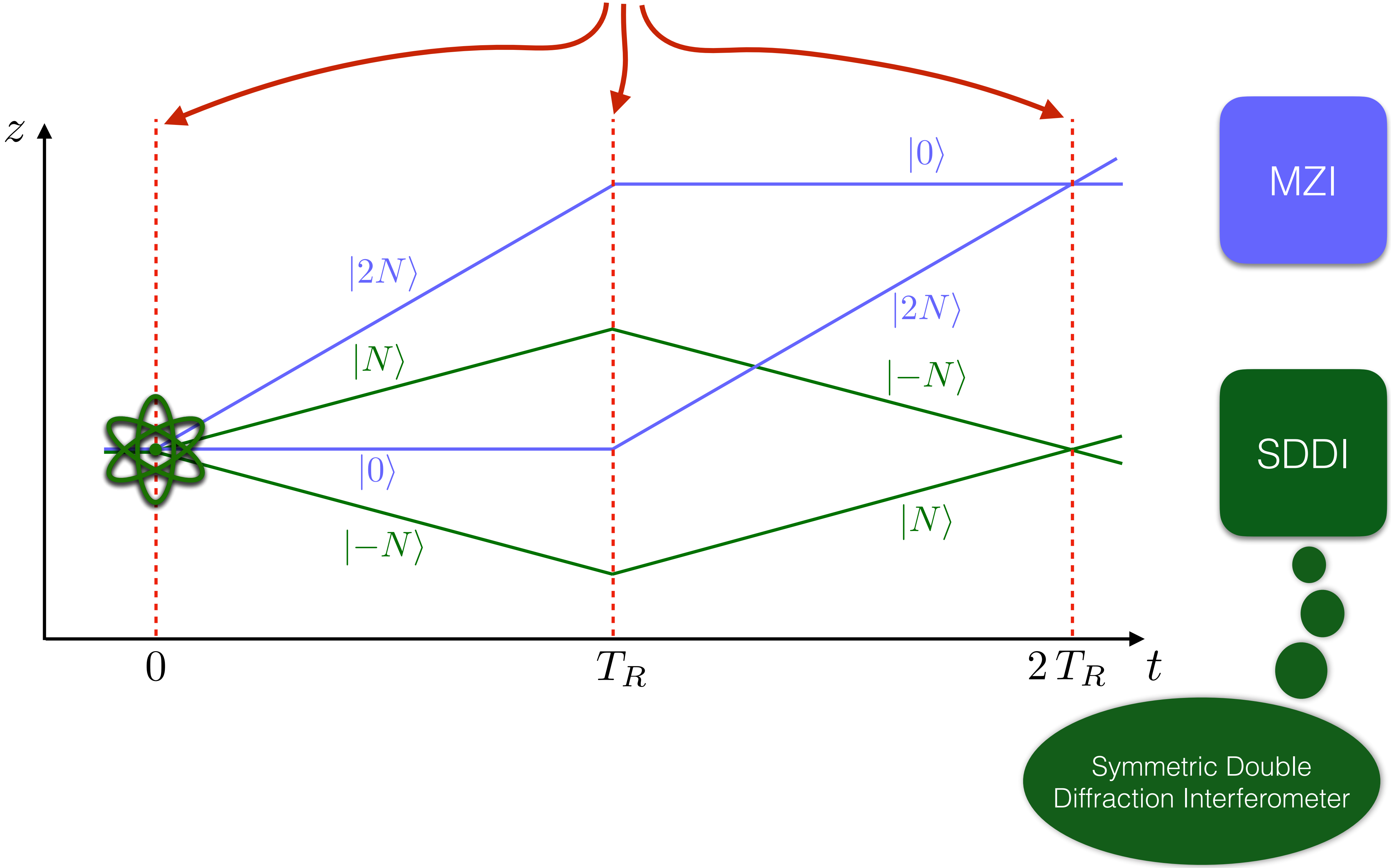
Estimator for gravity gradient: $\hat{\Gamma}_0 = \dfrac{\Delta g}{\Delta z}$

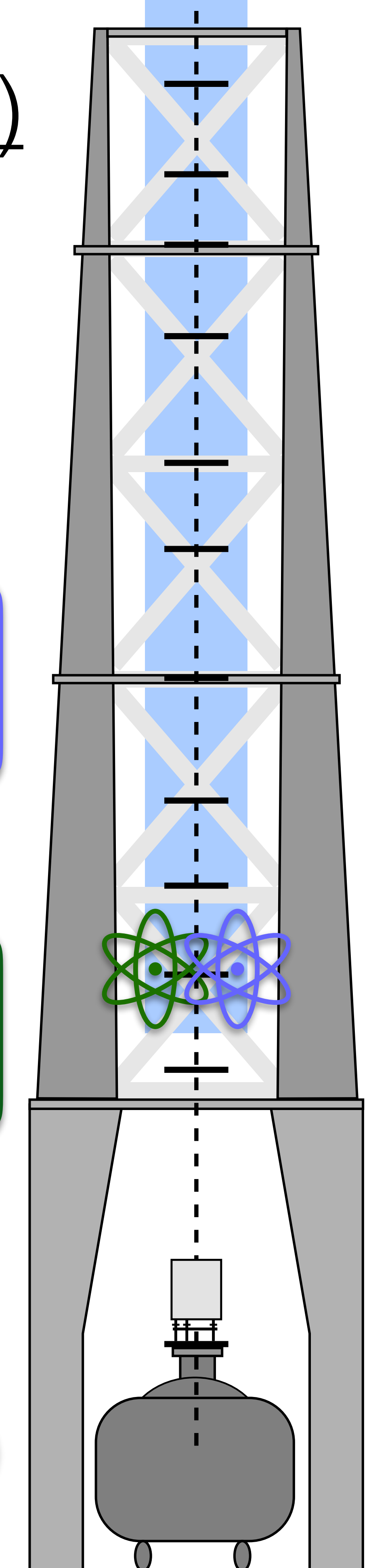


Co-located Gradiometric Interferometer (CGI)

We proposed a novel Interferometer geometry, the „CGI“.

For today: Set speed of light infinite. More details in the paper.





Phase shift of a CGI

Phase comparison of MZI and SDDI				
MZI	SDDI	Phase	Magnitude [rad]	Differential signal
2	2	$NkgT_R^2$	1.4×10^7	0
2	2	$Nkz_0\Gamma_0T_R^2$	20	0
2	2	$Nkv_0\Gamma_0T_R^3$	14	0
$-\frac{7}{6}$	$-\frac{7}{6}$	$Nkg\Gamma_0T_R^4$	14	0
2	0	$\frac{N^2\hbar k^2\Gamma_0T_R^3}{m}$	1.5×10^{-2}	2
-6	-6	$\frac{N\omega_R g^2T_R^3}{c^2}$	2.3×10^{-9}	0
6	6	$\frac{N\omega_R g v_0T_R^2}{c^2}$	2.4×10^{-9}	0

$$\Delta\Phi = 2\frac{N^2\hbar k_R^2\Gamma_0T_R^3}{m} = f \cdot \Gamma_0$$

$$f = 2\frac{N^2\hbar k_R^2T_R^3}{m}$$

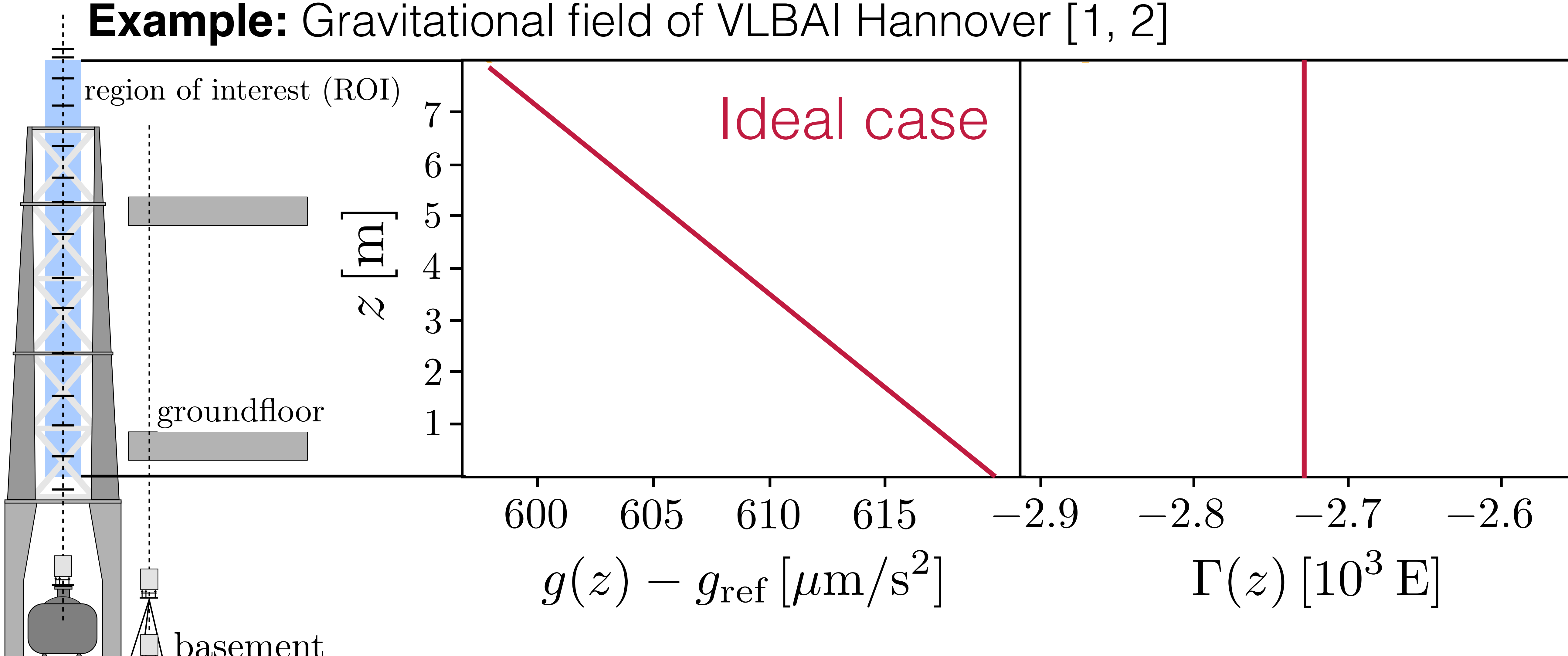
Answer 1: Using the CGI one can obtain knowledge about the gradient of the grav. field in a different way.

Arbitrary Ideal gravitational fields

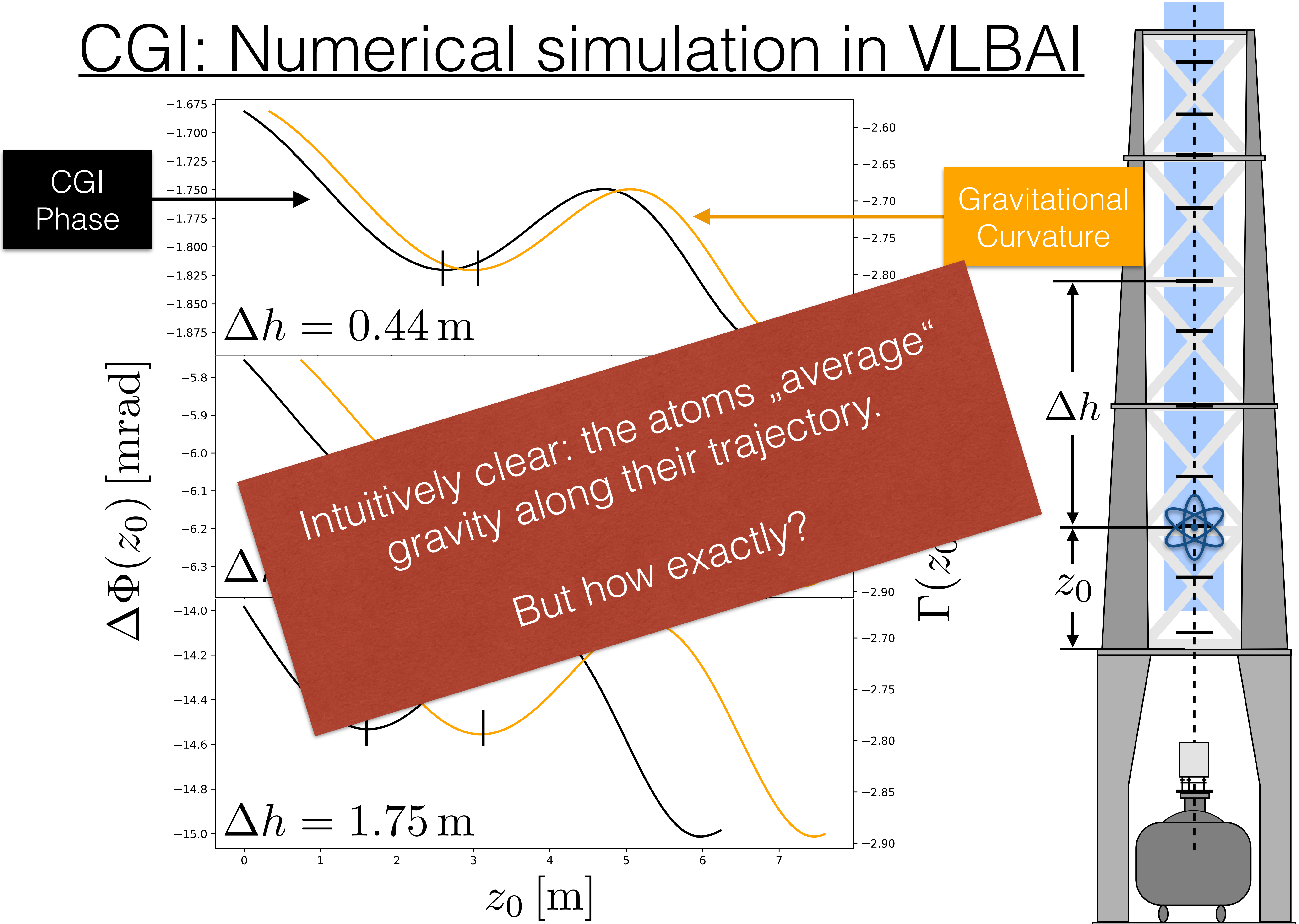
Consider $\phi(z) = \phi_0 + gz - \frac{1}{2}\Gamma_0 z^2$ with a constant gravity gradient.

with a non-trivial $\Gamma(z) = \partial_z^2 \phi(z)$ = Gravitational curvature

Example: Gravitational field of VLBAI Hannover [1, 2]



CGI: Numerical simulation in VLBAI



Intuitively clear: the atoms „average“ gravity along their trajectory.

But how exactly?

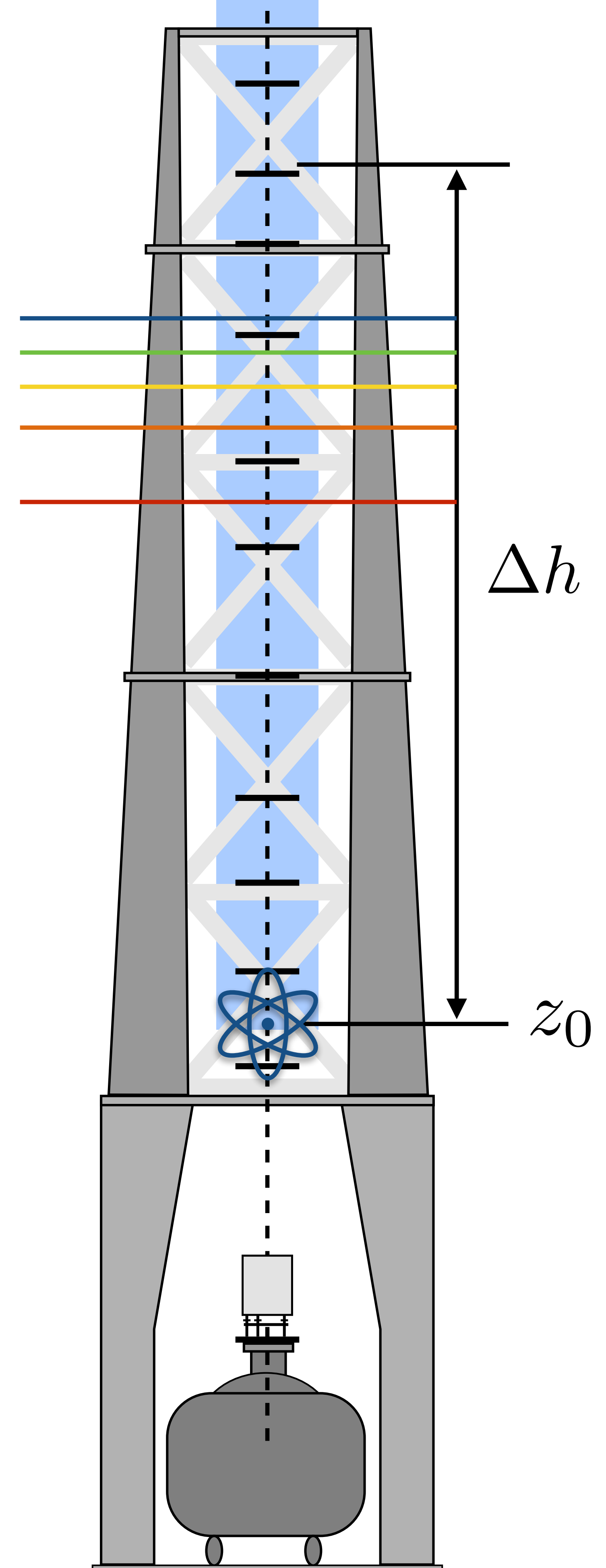
Averaging gravity

Where are the atoms „on average“?

Answer: It depends on the averaging process!

$$\|z(t)\|_n = \left(\frac{1}{2T_R} \int_0^{2T_R} |z(t) - z_0|^n \right)^{1/n}$$

n	1	2	3	4	5
$\|\|z(t)\|\|_n$	$0.66\,\Delta h$	$0.73\,\Delta h$	$0.77\,\Delta h$	$0.79\,\Delta h$	$0.82\,\Delta h$



Estimator for grav. curvature

We can now define a novel estimator $\hat{\Gamma}(z_0)$ for the grav. curvature:

Step 1: Convert phase to grav. curvature using the scale factor.

$$\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0)}{f}$$

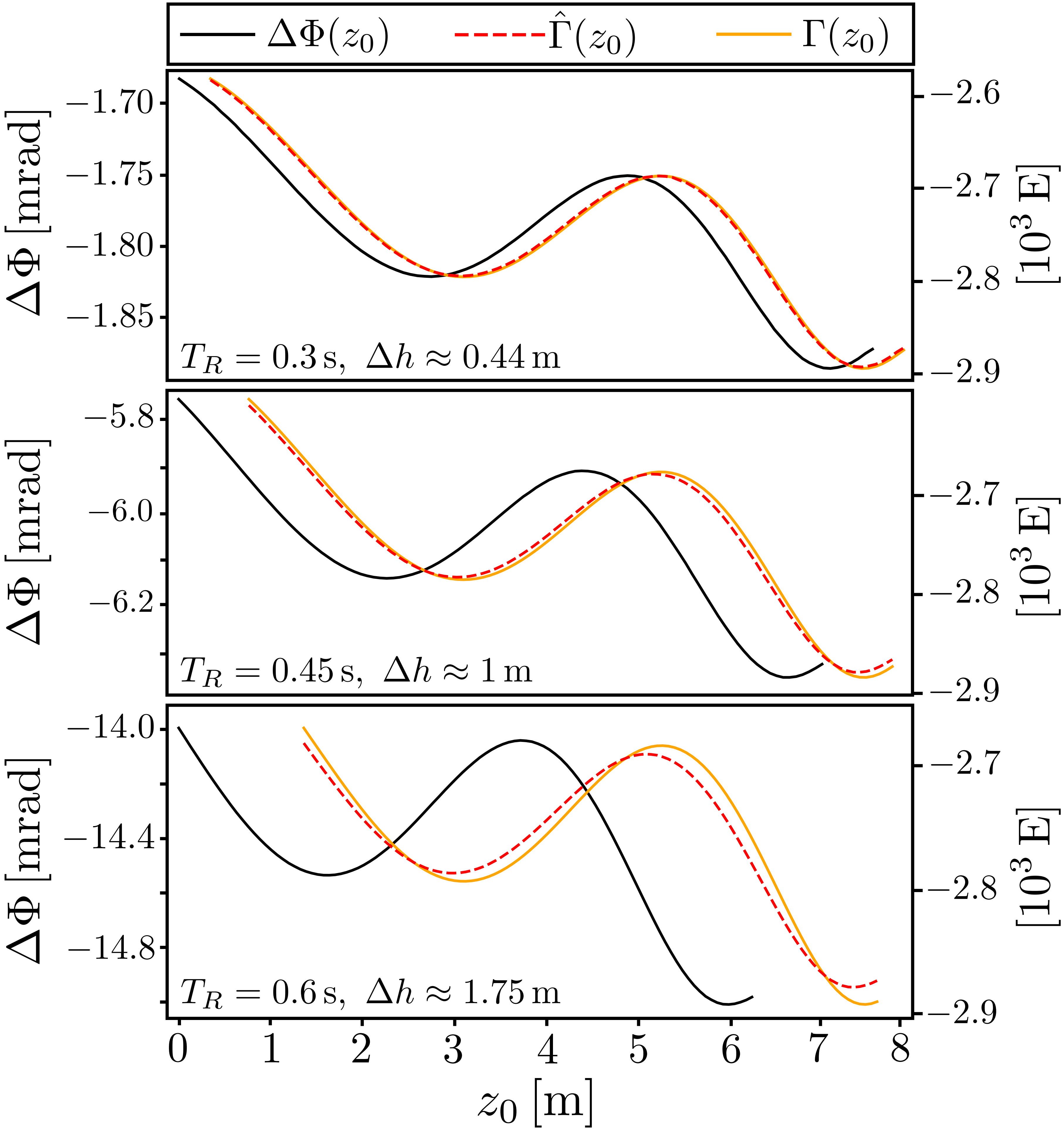
Step 2: Shift the height via the **cubic** mean of the trajectory.

$$\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0 - \|z(t)\|_3)}{f}$$

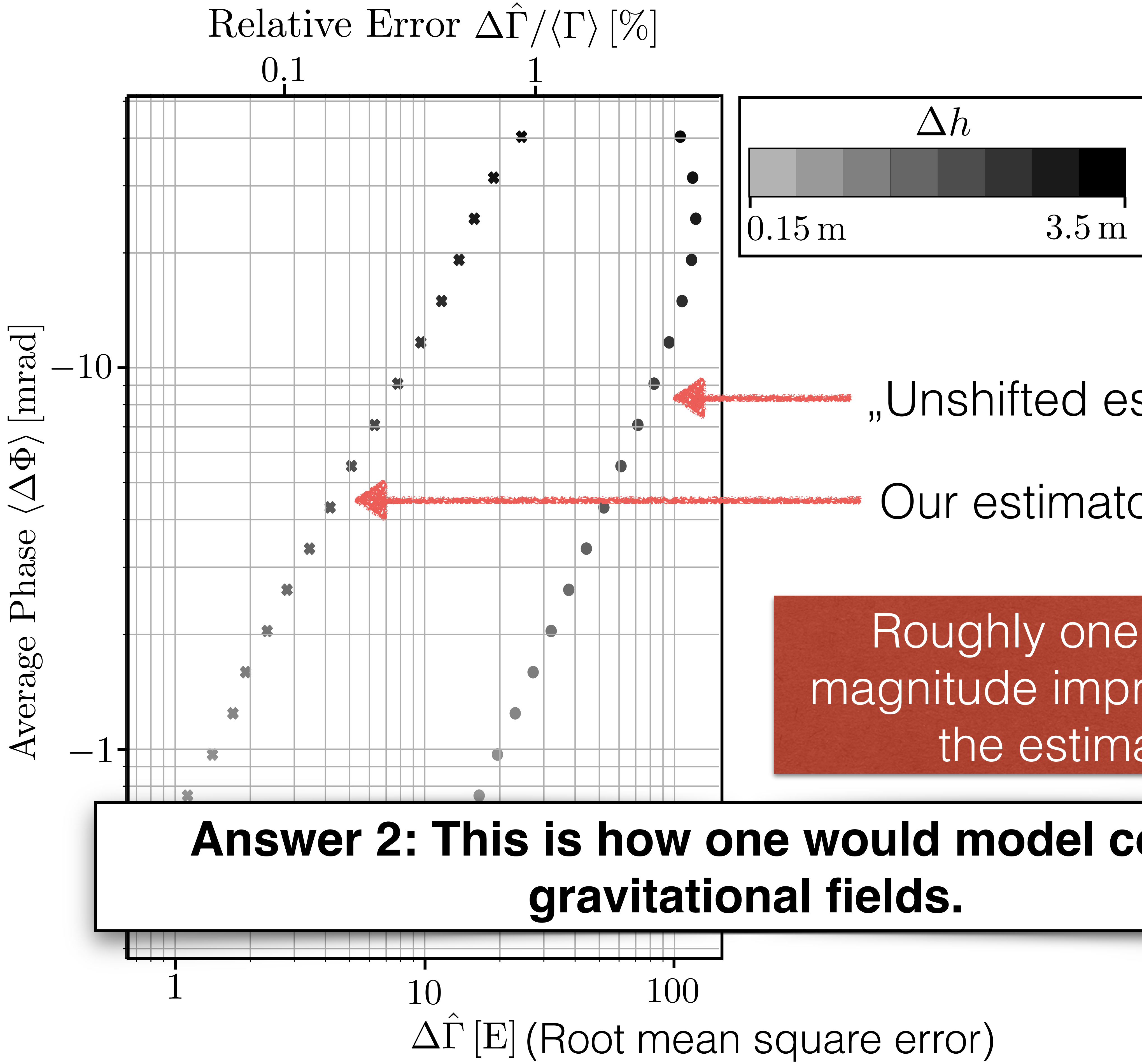
Reminder:

$$f = -2\frac{N^2 \hbar k_R^2 T_R^3}{m}$$

Motivation for the **cubic** mean.



$\Delta\Phi(z_0)$
$\hat{\Gamma}(z_0)$
$\Gamma(z_0)$
$\Delta\Phi$ [mrad]
[10^3 E]
$T_R = 0.3\,\mathrm{s},\ \Delta h \approx 0.44\,\mathrm{m}$
$T_R = 0.45\,\mathrm{s},\ \Delta h \approx 1\,\mathrm{m}$
$T_R = 0.6\,\mathrm{s},\ \Delta h \approx 1.75\,\mathrm{m}$
z_0 [m]
−1.70
−1.75
−1.80
−1.85
−5.8
−6.0
−6.2
−14.0
−14.4
−14.8
−2.6
−2.7
−2.8
−2.9
0
1
2
3
4
5
6
7
8



„Unshifted estimator“ $\frac{\Delta\Phi}{f}$

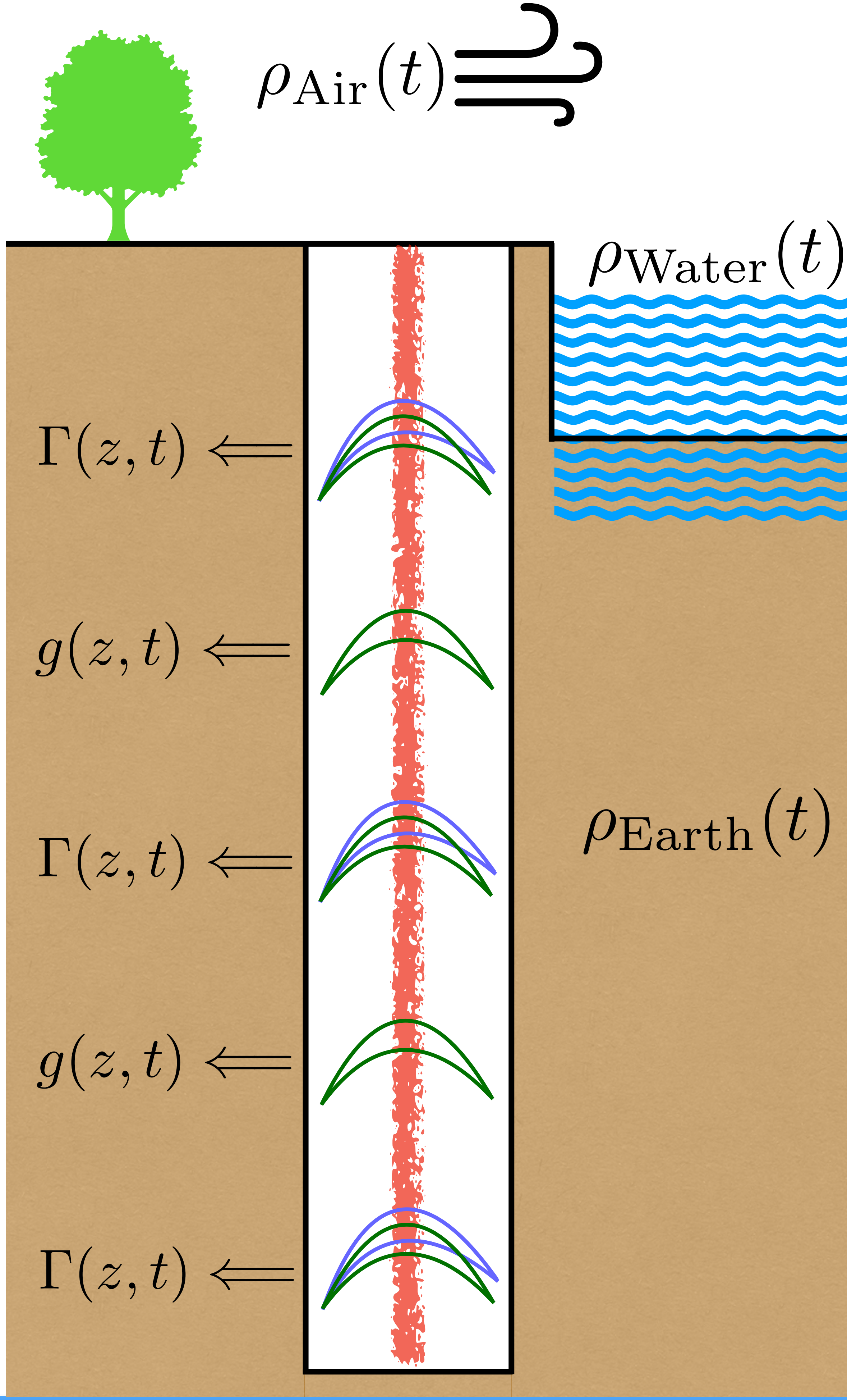
Our estimator $\hat{\Gamma}$

Roughly one order of magnitude improvement in the estimation!

Answer 2: This is how one would model complex gravitational fields.

Outlook / Summary

- We introduced the „CGI“ — giving us information about gravitational curvature.
- Gives rise to a novel way to measure gravitational field in large baseline interferometers.
- Possibility to measure the gravitational curvature using more compact devices.



Hammerer group

Thanks for listening!
Any questions?

Gaaloul group (T-SQUAD)

Link to the paper:

Link to the algorithm:

Other References:

- **[1]** Schilling et al. „Gravity field modelling for the Hannover 10m atom interferometer“

- **[2]** Lezeik et al. „Understanding the gravitational and magnetic environment of a very long baseline atom interferometer“

- **[3]** Albert Roura „Circumventing Heisenberg's Uncertainty Principle in Atom Interferometry Tests of the Equivalence Principle“

- **[4]** Amico et al. „Canceling the Gravity Gradient Phase Shift in Atom Interferometry“

Finite speed of light (FSL)

There are additional phase shifts, if one includes the FSL into the model!

Example: Two-Photon Bragg transitions would lead to an additional phase

$$\Delta\Phi_{\mathrm{FSL}} = \frac{4\hbar N^2 k^2 T_R}{mc}\left(4gT_R - v_0 - \frac{N\hbar k}{m}\right) + \Delta\Phi_0,$$

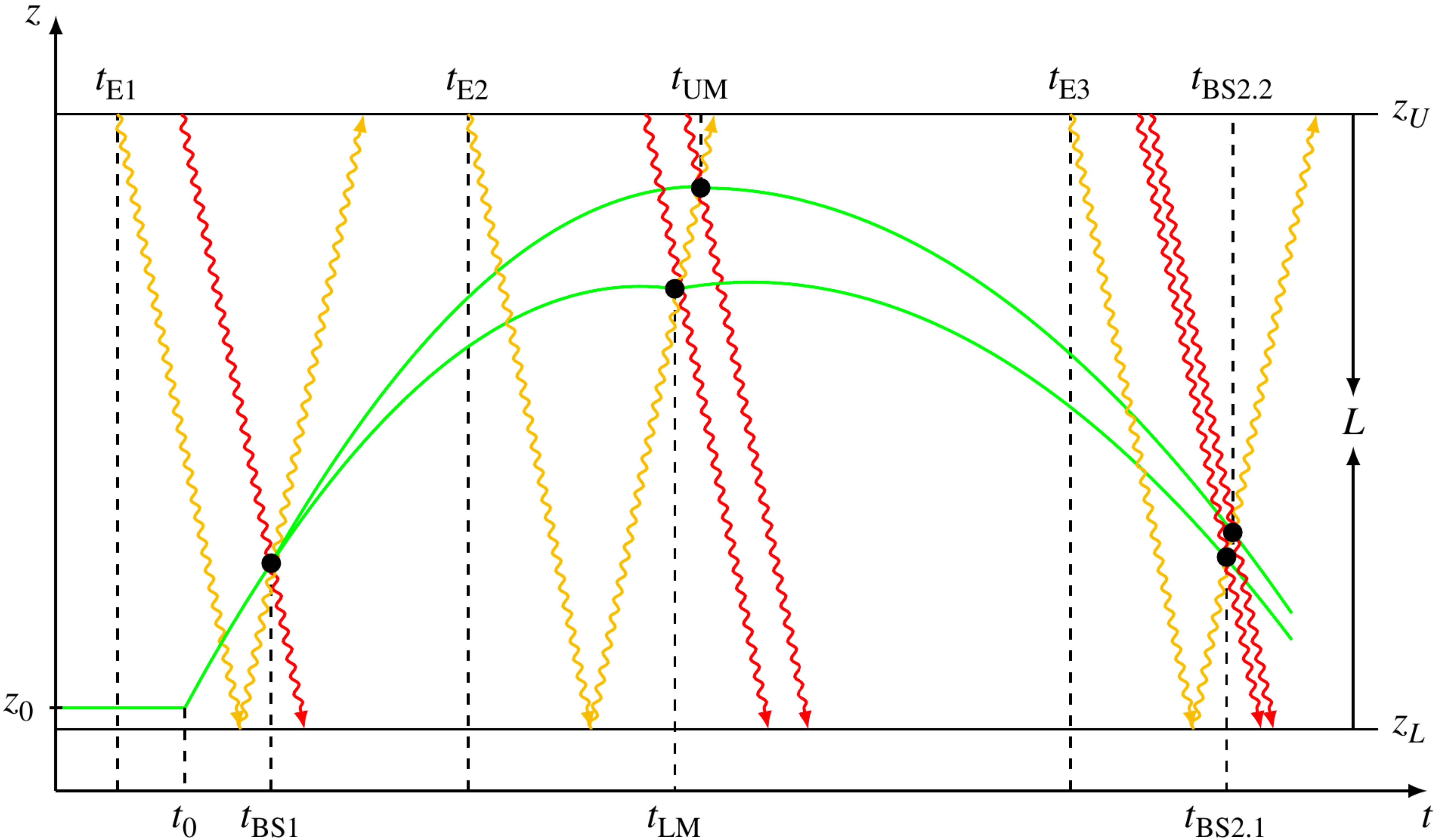
with a constant shift $\Delta\Phi_0 = \frac{2\hbar N^2 k^2}{mc}(2z_{\mathrm{L}} - z_0 - z_{\mathrm{U}}).$

z_{L}: Lower laser position

z_{U}: Upper laser position

Not that problematic,
since constant in time.

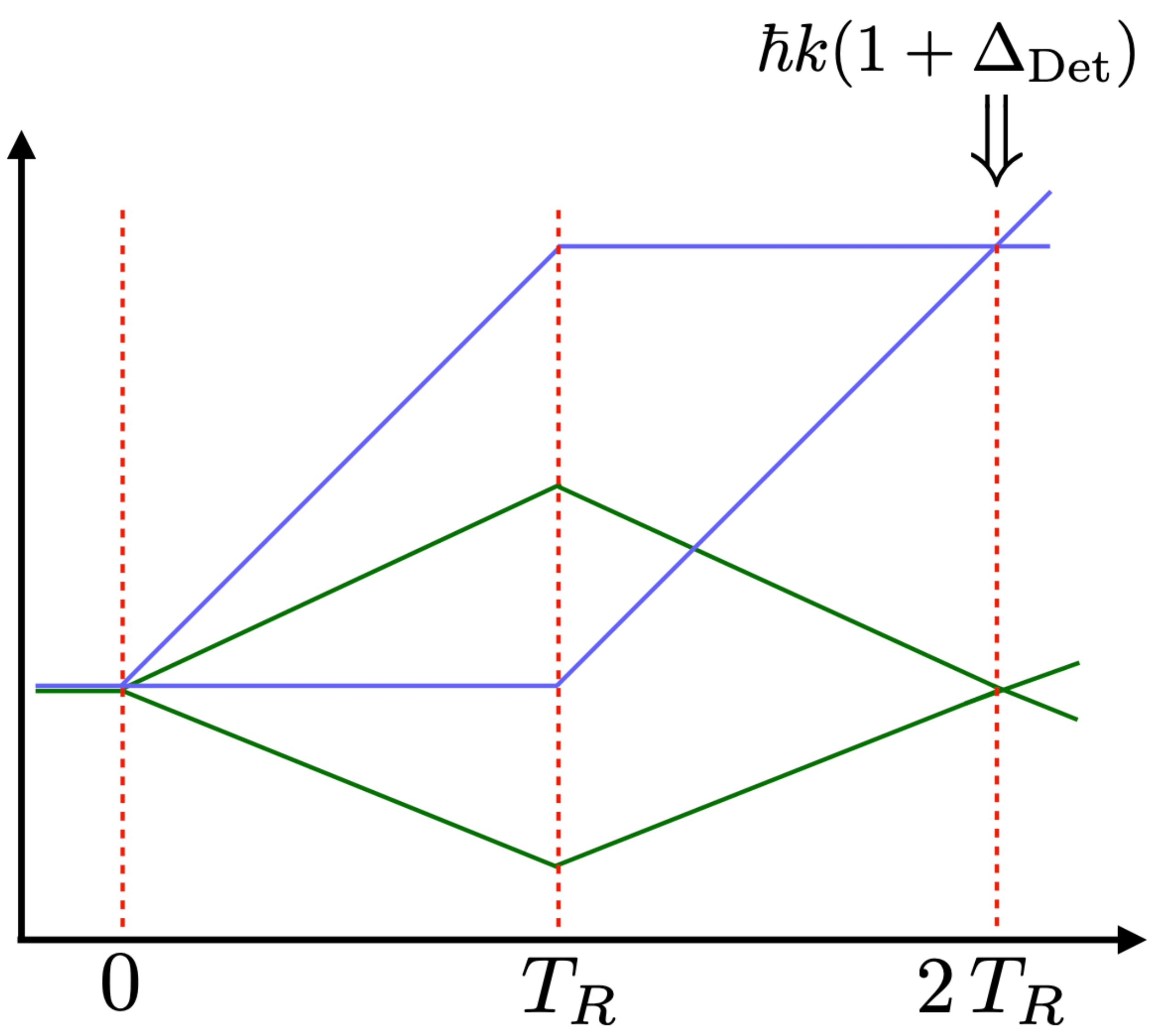
FSL with two-photon Bragg transitions



Photon paths of the example (FSL effect exaggerated).

FSL mitigation scheme

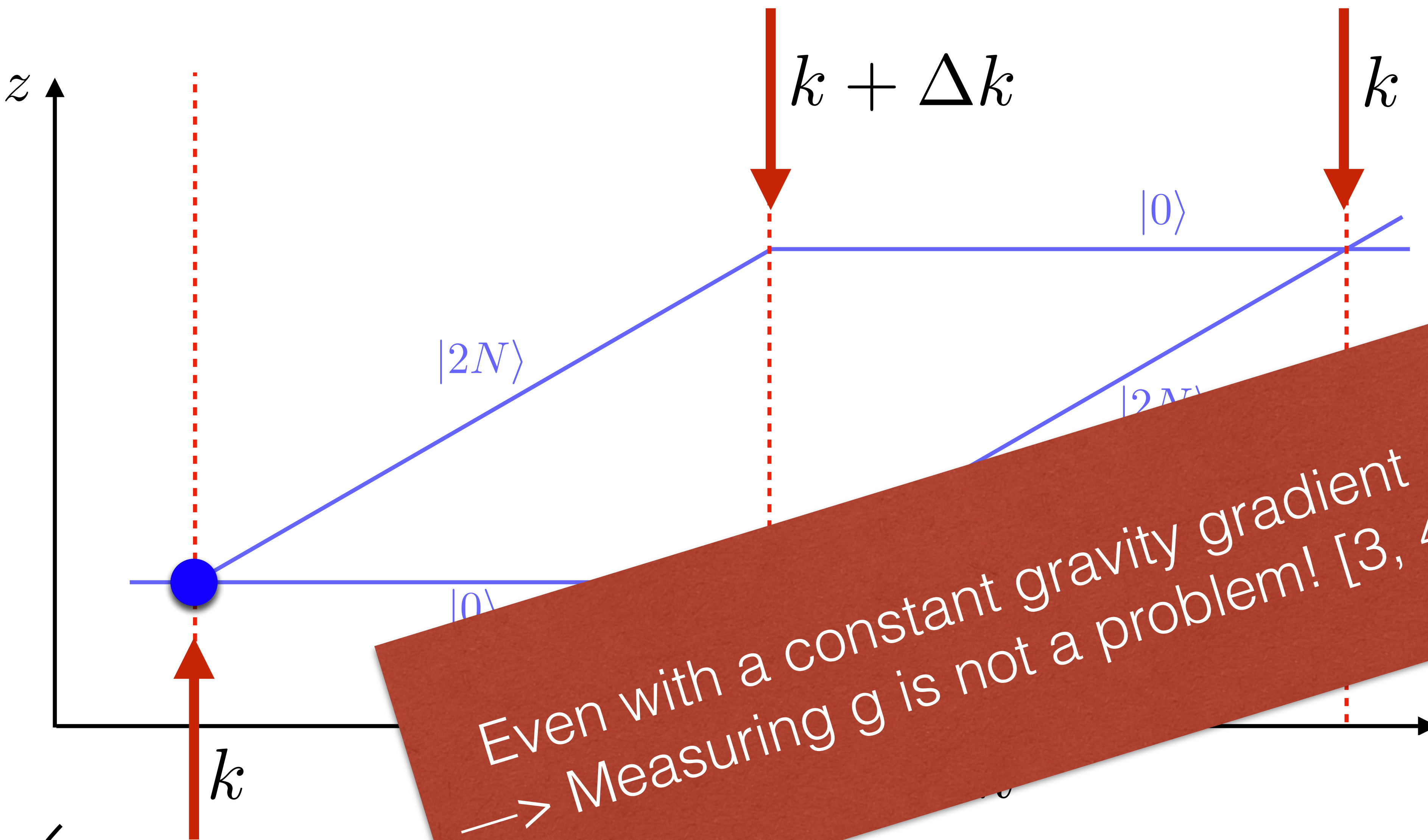
The time-dependent FSL phase can be mitigated!



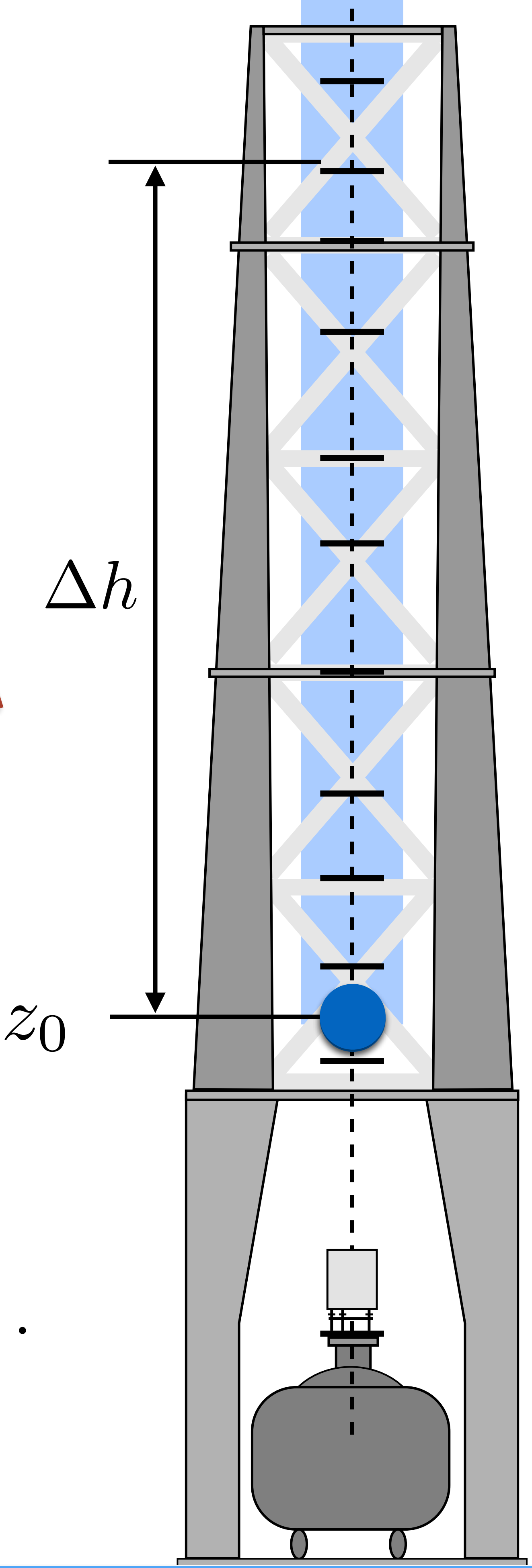
Alter the last AIF pulse by:

$$\Delta_{\mathrm{Det}}(v_0, T_R) = 2\frac{v_0 + \frac{\hbar k}{m} - 4gT_R}{v_0 + \frac{\hbar k}{m} - gT_R}\frac{\hbar k}{mc}$$

This frequency chirp ($\sim 100\,\mathrm{MHz}$) nullifies the FSL phase.



Even with a constant gravity gradient
—> Measuring g is not a problem! [3, 4]



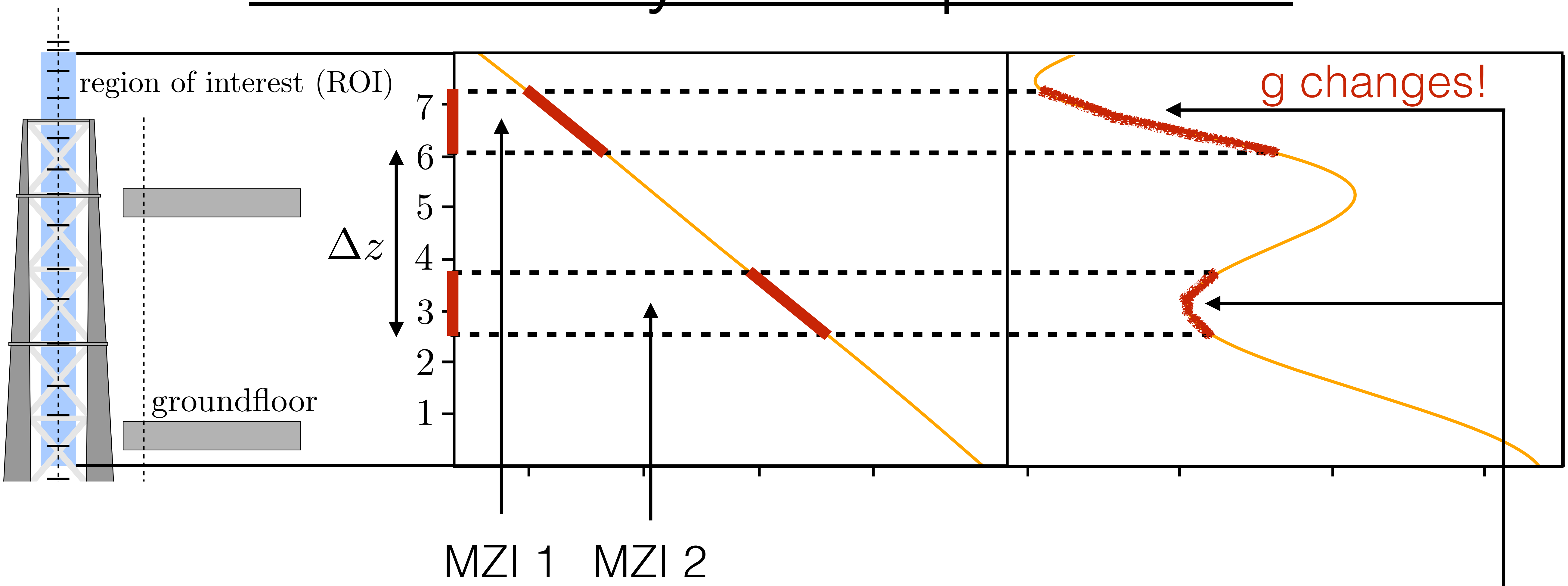
Phase shift:

$$\Delta k = \frac{1}{2}\Gamma_0 T_R^2 k$$

$$\Delta\Phi = 2NgkT_R^2 + \Gamma_0 Nk\underbrace{\left(2z_0T_R^2 + 2v_0T_R^3 - \frac{7}{6}gT_R^4\right)}_{\text{„Unwanted“ additional phases}} + \dots$$

„Unwanted“ additional phases

Gradiometry in complex fields



$$\hat{\Gamma}_0 = \frac{\Delta g}{\Delta z}$$

Each Mach-Zehnder Interferometer (MZI) measures „its“ g.

Divide change in g over the height difference.

If you want high spatial resolution —> Make Δz and Δh small

—> Higher measurement uncertainty