# Atom interferometers in weakly curved spacetimes using elastic scattering

## DPG Spring Meeting 2023 - SAMOP

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# Motivation

Atom interferometers are tools that can be to measure:

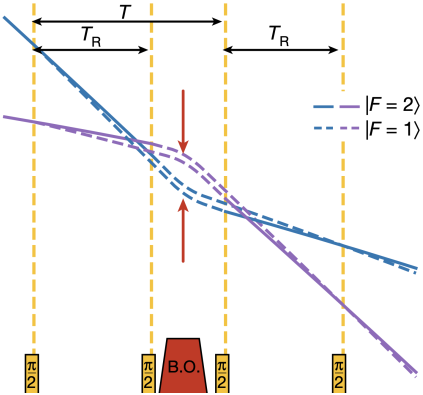
- the gravitational field $g$
- the gravitational gradient $\Gamma$
- atomic recoils and the fine structure constant $\alpha$
- inertial forces (accelerations, rotations, ...)

**Question:** How to include general relativity (GR) into a quantum mechanical system like this?

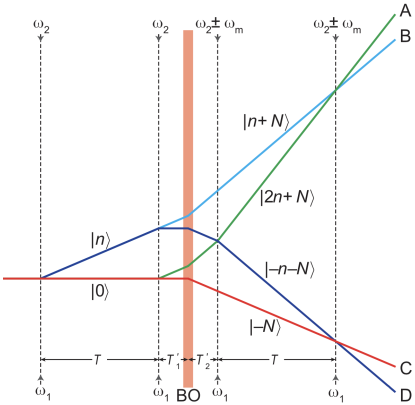
$\Longrightarrow$ Can atom interferometers be used to **test GR**?

# Atom Interferometers used to measure $\alpha$

Measurements of $\alpha$ have accuracies of some dozen parts per trillion!




Interferometer geometry from Morel et al. [1]




Interferometer geometry from Parker et al. [2]

# Possible Interferometer geometries:

We want to describe arbitrary interferometers of the following form:

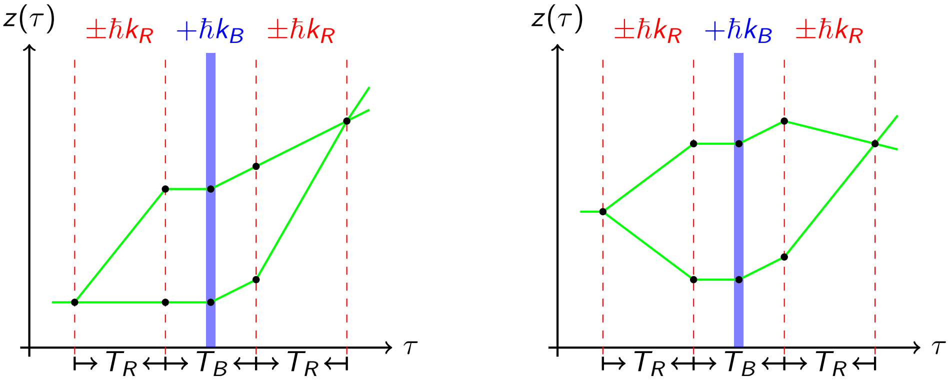



Figure: Example Geometries. Red lasers: Bragg scattering. Blue Laser: Bloch oscillations.

# Gravitational model

We model the space-time metric of the earth by

$$\mathrm{d}s^2 = -\left(c^2 + 2\phi(\vec{r}) + 2\beta\frac{\phi(\vec{r})^2}{c^2}\right)\mathrm{d}t^2 + \left(1 - 2\gamma\frac{\phi(\vec{r})}{c^2}\right)\mathrm{d}\vec{r}^2 + \mathcal{O}(c^{-4})$$

with 'test parameters' $\beta, \gamma$ (in GR: $\beta = \gamma = 1$) and the Newtonian gravitational potential $\phi(\vec{r})$.

We will approximate: $\phi(z) = \phi_0 + gz - \frac{1}{2}\Gamma z^2$, i.e. $\Gamma$ is the gravity gradient.

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$\Longrightarrow$ **Models spherically symmetric earth (non-rotating) with gravity gradient and relativistic effects to order $\mathcal{O}(c^{-2})$.**

# Free Propagation

Between the laser pulses: Solve the Schrödinger equation for this Hamiltonian

$$\hat{H}_{\mathrm{COM}} = mg\hat{Z} - \frac{m}{2}\Gamma\hat{Z}^2 + \frac{\hat{P}^2}{2m} + \frac{1}{mc^2}\left[\frac{2\gamma+1}{2}g\hat{P}\hat{Z}\hat{P} - \frac{\hat{P}^4}{8m^2} + \frac{2\beta-1}{2}m^2g^2\hat{Z}^2 + 2(\beta-1)m^2\phi_0 g\hat{Z}\right] + \mathcal{O}(\Gamma\, c^{-2}, c^{-4}). \tag{1}$$

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We use the stationary phase approximation using the corr. Lagrangian $L$, s.t. we obtain a phase difference along the paths:

$$\Delta\Phi_{\mathrm{Prop}} := \frac{1}{\hbar}\int [L(z_{\mathrm{u}}(t)) - L(z_{\mathrm{l}}(t))]\,\mathrm{d}t. \tag{2}$$

**Task:** Solve the Euler-Lagrange equation to get $z_{\mathrm{u/l}}(t)$ and perform the integral.

# Electromagnetism

**EM-Lagrange function:** $L_{\mathrm{EM}} = \int -\frac{\sqrt{-g}}{4\mu_0} F_{\mu\nu} F^{\mu\nu} \mathrm{d}^3\vec{x}$ with $g = \det(g_{\mu\nu})$.

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**Solution:** $\vec{A} = \mathcal{A}\left(1 - (\gamma + 1)\frac{gz}{c^2}\right)\vec{\varepsilon}\, e^{i\Phi(z,t)} + \mathcal{O}(\Gamma\, c^{-2})$.

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The phase is given by $\Phi(z,\tau) = \omega t \pm \left(1 - \frac{\gamma+1}{2}\frac{gz}{c^2}\right) k_z z + \mathcal{O}(\Gamma\, c^{-2})$.

# Interaction Hamiltonian

The interaction Hamiltonian will be modeled by the dipole Hamiltonian and the 'Röntgen term'

$$\hat{H}_{\text{A-L}} = -\vec{d}\cdot\vec{E}(\hat{Z}) + \frac{1}{2m}\left[\vec{P}\cdot\left(\vec{d}\times\vec{B}(\hat{Z})\right) + \text{h.c.}\right].$$

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$\Longrightarrow$ Analyze this for **elastic scattering** processes, i.e. Bragg scattering and Bloch oscillations.

$\Longrightarrow$ Calculate imprinted phase during laser interactions.

# Python Code

We programmed a Python code to quickly calculate all relevant terms to order $\mathcal{O}(c^{-2})$.

- Describe time decomposition, e.g. $(T_R, T_B, T_R)$.
- How many momentum quanta are in each path segment, e.g. upper path $[0, 1, 0]$.

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$\implies$ The code only leaves out the 'Finite Speed of Light' (FSL) effects.

$\implies$ I can show it to you now.

# References

Léo Morel, Zhibin Yao, Pierre Cladé, and Saïda Guellati-Khélifa.
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