Local Measurement of Gravitational Curvature using Atom Interferometers

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CRC 1227 DQ-mat Designed Quantum States of Matter

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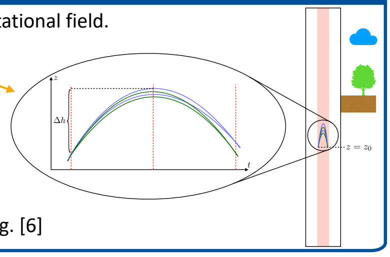
Introduction & Motivation

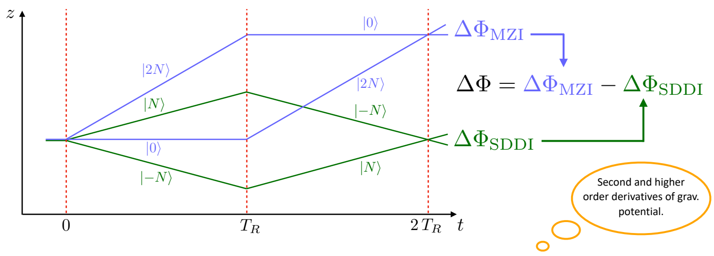
- Light pulse atom interferometers (AIF) are exquisite quantum probes of the spatial inhomogeneity and curvature of the gravitational field.
- We present a geometry, the ‚Co-located Gradiometric Interferometer' (CGI), in which the differential signal of two co-located interferometers singles out a phase proportional to gravitational curvature. [1]
- As a case study, we theoretically examine the implications and implementation of this CGI geometry in the context of the Hannover VLBAI facility. [2]
- Measuring the gravitational curvature with quantum systems has recently gained interest:
  - → Quantum mechanical measurements of spacetime curvature [3] or gravitational entanglement. [4]
  - → The proposed ‚Gravitational Aharonov Bohm effect'. [5]
  - → Detecting gravitational anharmonicities in civil engineering. [6]



Co-located Gradiometric Interferometer (CGI)

We define the CGI as a differential measurements scheme between to AIFs initialized at the same height [1].

The CGI consists of a Mach-Zehnder Interferometer (MZI) and a Symmetric Double Diffraction Interferometer (SDDI) like this:



The differential phase of this AIF is especially sensitive to **gravitational curvature**.

Phase Shift in Idealized Gravitational Fields

Consider an idealized gravitational potential as: $\phi_{\mathrm{Ideal}}(z) = gz + \frac{1}{2}\Gamma_0 z^2$

Earth's gravity gradient $\Gamma_0 \approx -3 \times 10^3\,\mathrm{E}$

Calculating the phases in this scenario leads to:

Phase comparison of MZI and SDDI

MZI	SDDI	Phase	Magnitude [rad]	Differential signal
2	2	$NkgT_R^2$	1.4×10^7	0
2	2	$Nkz_0\Gamma_0 T_R^2$	20	0
2	2	$Nkv_0\Gamma_0 T_R^3$	14	0
$-\frac{7}{6}$	$-\frac{7}{6}$	$Nkg\Gamma_0 T_R^4$	14	0
2	0	$\frac{N^2\hbar k^2\Gamma_0 T_R^3}{m}$	1.5×10^{-2}	2
−6	−6	$\frac{N\omega_R g^2 T_R^3}{c^2}$	2.3×10^{-9}	0
6	6	$\frac{N\omega_R g v_0 T_R^2}{c^2}$	2.4×10^{-9}	0
10	0	$\frac{N^2\omega_R\hbar k g T_R^2}{mc^2}$	1.1×10^{-12}	10
−4	0	$\frac{N^2\omega_R\hbar k v_0 T_R}{mc^2}$	1.1×10^{-12}	−4
0	4	$\frac{N^3\omega_R\hbar^2 k^2 T_R}{m^2c^2}$	5.7×10^{-16}	−4

Not included in this list: Finite speed of light (FSL) contributions. Mitigated below anyway.

Phase dominantly given by:

$$\Delta\Phi \approx \frac{2\Gamma_0 N^2\hbar k^2 T_R^3}{m} = f \cdot \Gamma_0$$

Scale factor f is known to high precision!

Only depends on:

k (Laser wavenumber), $\hbar/m$ (Atomic recoil), T_R (AIF time)

Same as ‚tidal phase' in [3, 5] but with different origin.

Additional phase shift contributions are very small

FSL Phase Mitigation

The FSL phases heavily depend on the experimental setup. As an example consider two-photon Bragg scattering processes:

Resulting FSL phase $$\Delta\Phi_{\mathrm{FSL}} = \frac{4\hbar N^2 k^2 T_R}{mc}\left(4gT_R - v_0 - \frac{N\hbar k}{m}\right) + \Delta\Phi_0$$ (const.)

Mitigation by common frequency chirp at the third IF pulse of ν_{Det}.

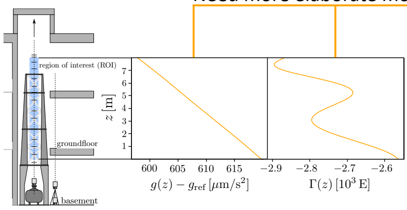
$\Rightarrow$ Additional phase $\Delta\Phi_{\mathrm{Additional}} = 2NT_R\nu_{\mathrm{Det}}\frac{v_0 + \frac{N\hbar k}{m} - gT_R}{c}$ can be tuned to cancel the FSL phase. Typical order of magnitude (10m): $\nu_{\mathrm{Det}} \approx 100\,\mathrm{MHz}$

Real Gravitational Field: VLBAI Hannover

Need more elaborate model

$$\phi(z) = g\,z + \underbrace{\sum_{n=2} \frac{\phi^{(n)}}{n!} z^n}_{= \Gamma(z)}$$

Grav. curvature



Gravitational acceleration $g(z)$ and gravitational gradient $\Gamma(z)$ as functions of height in the region of interest (ROI) of VLBAI Hannover. $g(z)$ is interpolated by a polynomial fit. Building cross-section taken from [7] and adapted.

How does the phase shift look like in this grav. field?

Extending the analysis to this more complicated grav. field we see that

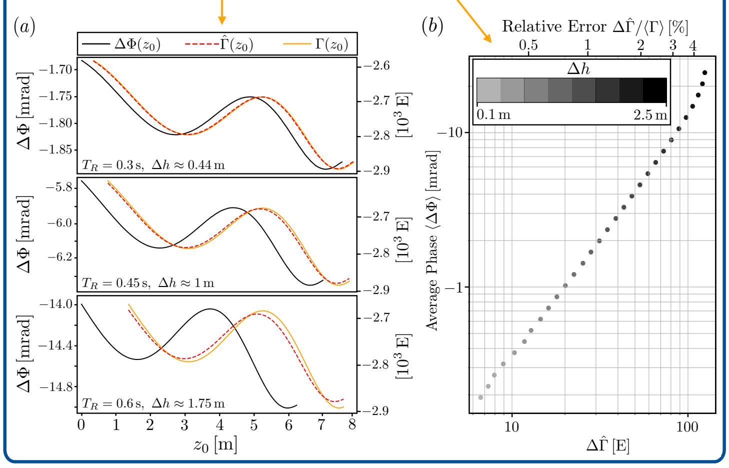
$$\Delta\Phi \approx -\frac{m}{\hbar}\sum_{n=2}\frac{\phi^{(n)}}{n!}\left[\mathcal{A}_{\mathrm{MZI}}(n) - \mathcal{A}_{\mathrm{SDDI}}(n)\right]$$

with $\mathcal{A}_{\mathrm{MZI}}(n) = \int_0^{2T_R}\left(z_{\mathrm{up}}^{\mathrm{MZI}}(t)^n - z_{\mathrm{low}}^{\mathrm{MZI}}(t)^n\right)\mathrm{d}t\,.$ (Classical atomic trajectories.)

The CGI is robust under the transition to complex grav. fields and we show how one can define an estimator for the grav. curvature from the phase measurement.

Estimator $\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0 - \|z(t)\|_3)}{f}$ with $\|z(t)\|_3 = \left(\frac{1}{2T_R}\int_0^{2T_R}|z(t) - z_0|^3\,\mathrm{d}t\right)^{1/3}$. (Cubic mean. Roughly $0.77\Delta h$)

Phase shift describes grav. curvature at a height roughly 77% of Δh higher. Scale factor f can still be used as a good estimator for grav. curvature [1, 2].



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