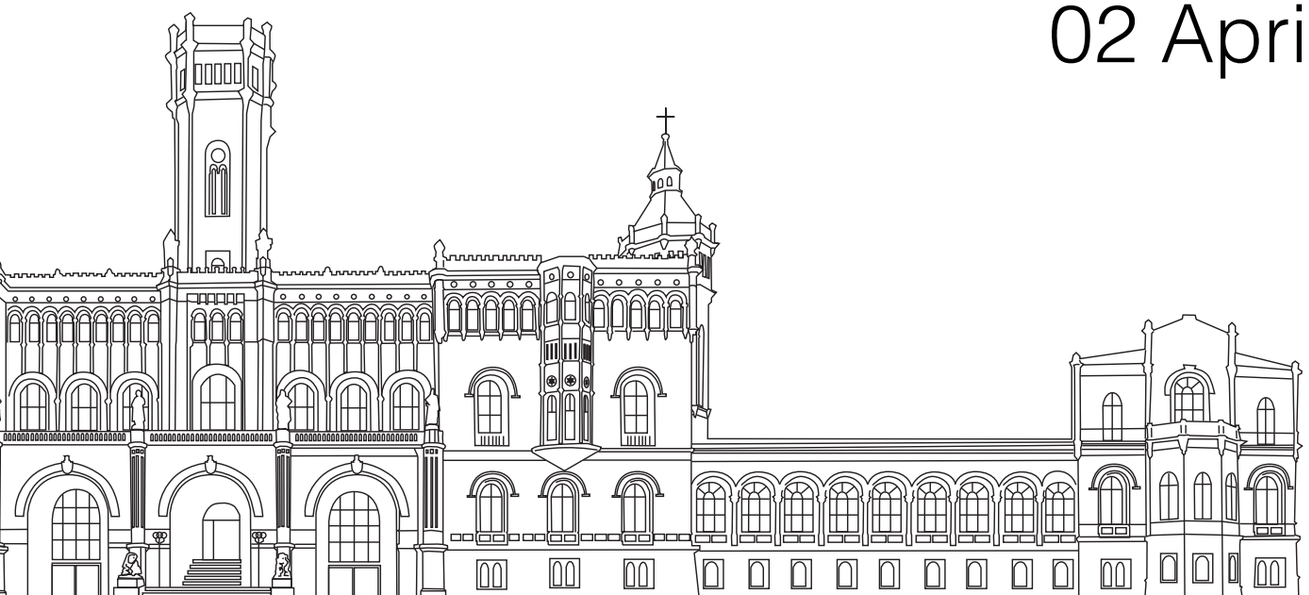
Leibniz Universität Hannover

Systematic analysis of atom interferometric measurements in complex gravitational environments

Michael Werner
Institute of Quantum Optics Hannover



02 April 2025

@ Rencontres de Moriond 2025 — Gravitation

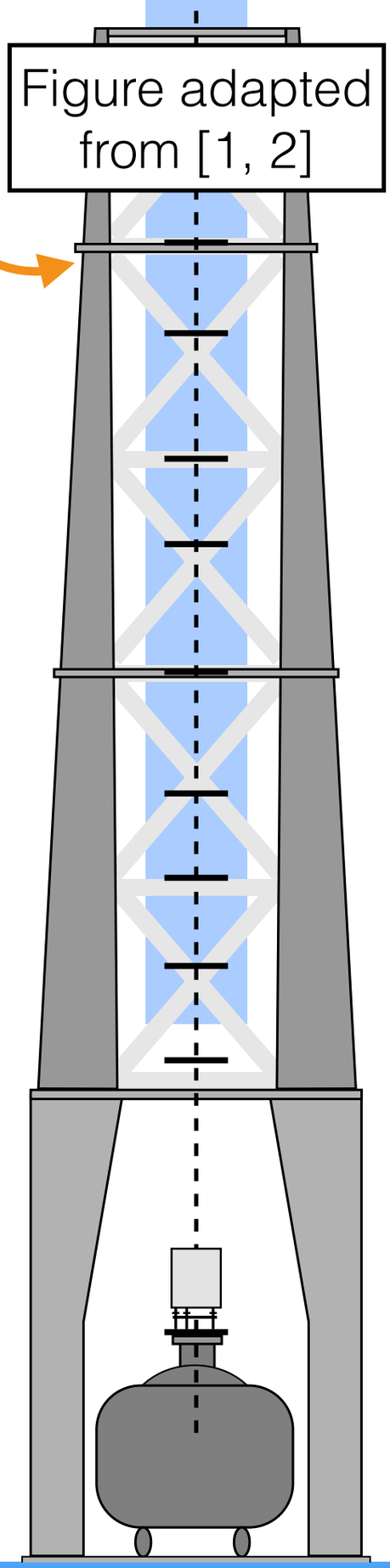
Motivation

Atom interferometers like the **VLBAI in Hannover** are highly accurate sensors for the gravitational field and **span about 10 meters**.

New atom interferometers are planned with **baselines of more than 100 meters**.

And even **atom interferometers in space** have been proposed.

Question: How do you model complex gravitational fields and even general relativistic effects in such quantum systems?



PPN Hamiltonian

Our theory colleagues working on general relativity Domenico Giulini and Philip Schwartz [3] developed a Hamiltonian describing:

Two point particles (think of Proton and Electron):

1. Coupled gravitationally to the PPN spacetime
2. Coupled electromagnetically to each other
3. EM field coupling to gravity

(Parametrized) Post-Newtonian (PPN) spacetime

$$g_{\mu\nu} = \begin{pmatrix} -1 - 2\frac{\phi}{c^2} - 2\beta\frac{\phi^2}{c^4} & 0 \\ 0 & \left(1 - 2\gamma\frac{\phi}{c^2}\right)\mathbb{1} \end{pmatrix}$$

PPN Hamiltonian

$$
\begin{aligned}
H_{\mathrm{C,final}} &= \frac{\mathbf{P}^2}{2M}\left[1-\frac{1}{Mc^2}\left(\frac{\mathbf{p_r}^2}{2\mu}+\frac{e_1e_2}{4\pi\varepsilon_0 r}\right)\right]+\left[M+\frac{1}{c^2}\left(\frac{\mathbf{p_r}^2}{2\mu}+\frac{e_1e_2}{4\pi\varepsilon_0 r}\right)\right]\phi(\mathbf{R}) \\
&\quad -\frac{\mathbf{P}^4}{8M^3c^2}+\frac{2\gamma+1}{2Mc^2}\mathbf{P}\cdot\phi(\mathbf{R})\mathbf{P}+(2\beta-1)\frac{M\phi(\mathbf{R})^2}{2c^2}\ , \\
H_{\mathrm{A,final}} &= \frac{{}^{(3)}g^{-1}_{\mathbf{R}}(\mathbf{p_r},\mathbf{p_r})}{2\mu}+\frac{e_1e_2}{4\pi\varepsilon_0\sqrt{{}^{(3)}g_{\mathbf{R}}(\mathbf{r},\mathbf{r})}} \\
&\quad -\frac{m_1^3+m_2^3}{M^3}\frac{\mathbf{p_r}^4}{8\mu^3c^2}+\frac{e_1e_2}{4\pi\varepsilon_0}\frac{1}{2\mu Mc^2}\left(\mathbf{p_r}\cdot\frac{1}{r}\mathbf{p_r}+\mathbf{p_r}\cdot\mathbf{r}\frac{1}{r^3}\mathbf{r}\cdot\mathbf{p_r}\right)\ , \\
H_{\mathrm{AL,final}} &= -\mathbf{d}\cdot\mathbf{E}^{\perp}_{\mathrm{coord.}}(\mathbf{R})+\frac{1}{2M}\{\mathbf{P}\cdot[\mathbf{d}\times\mathbf{B}_{\mathrm{coord.}}(\mathbf{R})]+\mathrm{H.c.}\} \\
&\quad -\frac{m_1-m_2}{4m_1m_2}\{\mathbf{p_r}\cdot[\mathbf{d}\times\mathbf{B}_{\mathrm{coord.}}(\mathbf{R})]+\mathrm{H.c.}\} \\
&\quad +\frac{1}{8\mu}(\mathbf{d}\times\mathbf{B}_{\mathrm{coord.}}(\mathbf{R}))^2+\frac{1}{2\varepsilon_0}\int \mathrm{d}^3\mathbf{x}\left(1+(\gamma+1)\frac{\phi}{c^2}\right){\boldsymbol{\mathcal{P}}^{\perp}_d}^2(\mathbf{x},t)\ , \\
H_{\mathrm{L,final}} &= \frac{\varepsilon_0}{2}\int \mathrm{d}^3\mathbf{x}\left(1+(1-3\gamma)\frac{\phi}{c^2}\right)\left[{\mathbf{E}^{\perp}_{\mathrm{phys.}}}^2+c^2\mathbf{B}^2_{\mathrm{phys.}}\right] \\
&= \frac{\varepsilon_0}{2}\int \mathrm{d}^3\mathbf{x}\sqrt{-g}\left[{\mathbf{E}^{\perp}_{\mathrm{phys.}}}^2+c^2\mathbf{B}^2_{\mathrm{phys.}}\right]
\end{aligned}
$$

What we did with this:

We applied this Hamiltonian to atom interferometers.

Created a Python algorithm that **symbolically** calculates the phase shifts (using “SymPy”).

Made it open source for everyone to use it!

Theoretical model:

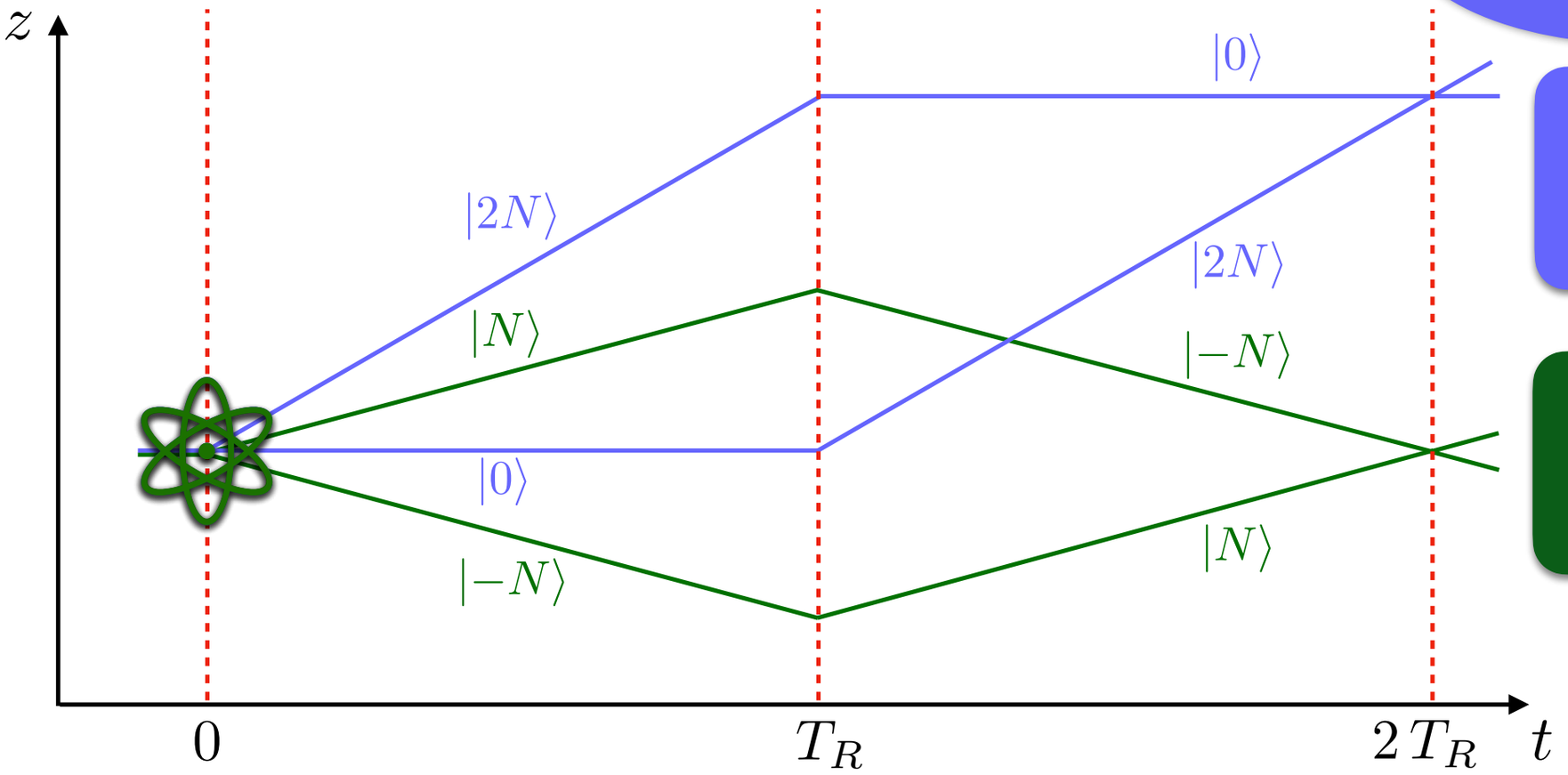
Link to the algorithm:

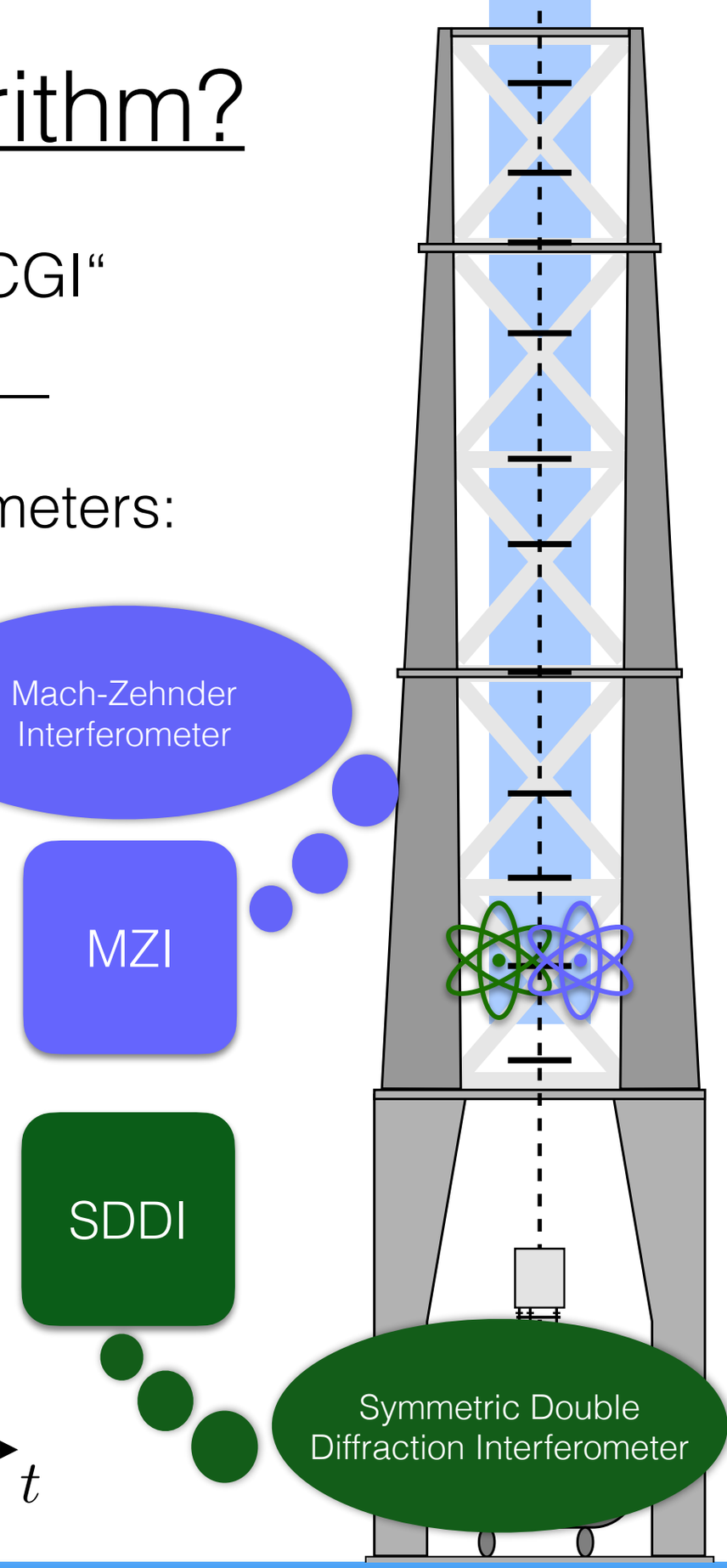
What did we do with this algorithm?

We proposed a novel interferometer geometry, the „CGI“

— “**Co-located Gradiometric Interferometer**” —

as a differential measurement between two interferometers:





(Leading) Phase shift of a CGI

Phase comparison of MZI and SDDI

MZI	SDDI	Phase	Magnitude [rad]	Differential signal
2	2	$NkgT_R^2$	1.4×10^7	0
2	2	$Nkz_0\Gamma_0 T_R^2$	20	0
2	2	$Nkv_0\Gamma_0 T_R^3$	14	0
$-\frac{7}{6}$	$-\frac{7}{6}$	$Nkg\Gamma_0 T_R^4$	14	0
2	0	$\frac{N^2\hbar k^2\Gamma_0 T_R^3}{m}$	1.5×10^{-2}	2
−6	−6	$\frac{N\omega_R g^2 T_R^3}{c^2}$	2.3×10^{-9}	0
6	6	$\frac{N\omega_R g v_0 T_R^2}{c^2}$	2.4×10^{-9}	0

$$\Delta\Phi = 2\frac{N^2\hbar k_R^2\Gamma_0 T_R^3}{m} = f \cdot \Gamma_0$$

$$f = 2\frac{N^2\hbar k_R^2 T_R^3}{m}$$

Gravitational potential: $\phi(z) = \phi_0 + gz - \frac{1}{2}\Gamma_0 z^2 + \mathcal{O}(z^3)$

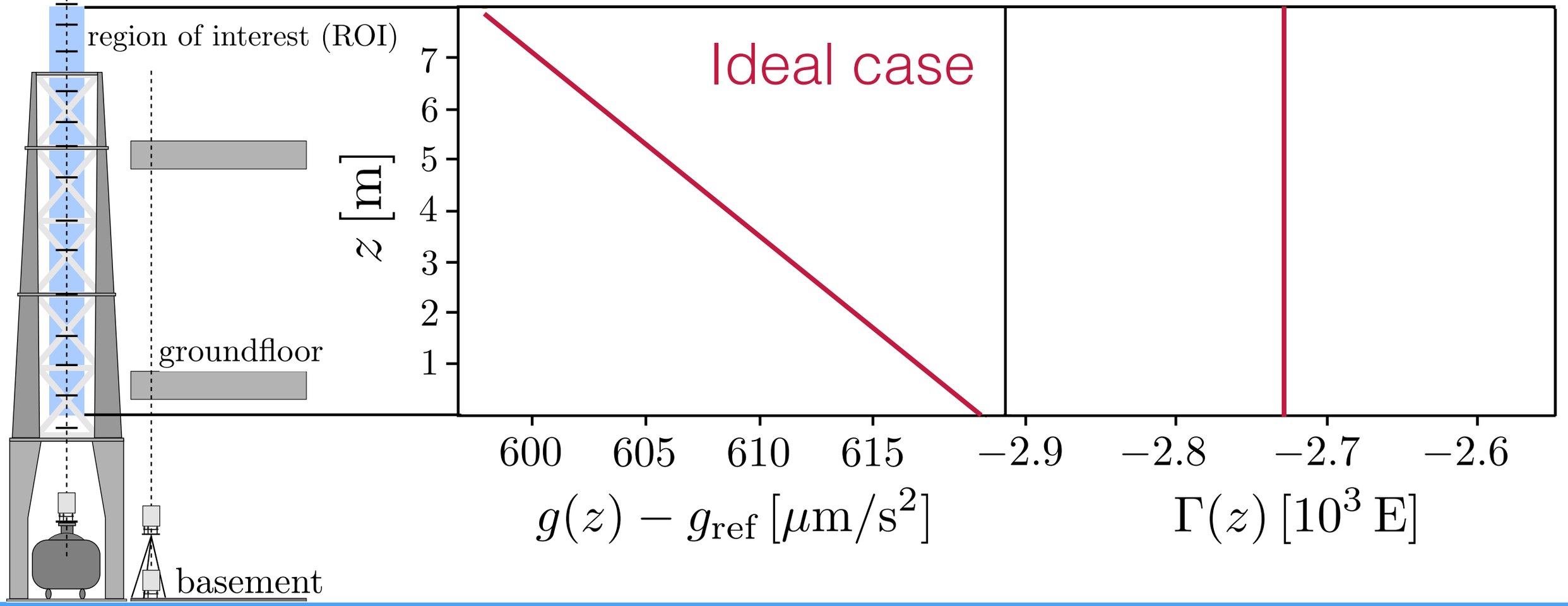
Arbitrary gravitational fields

Ideal

Consider $\phi(z) = \phi_0 + gz - \frac{1}{2}\Gamma_0 z^2$ with a constant gravity gradient.

with a non-trivial $\Gamma(z) = \partial_z^2 \phi(z)$ = Gravitational curvature

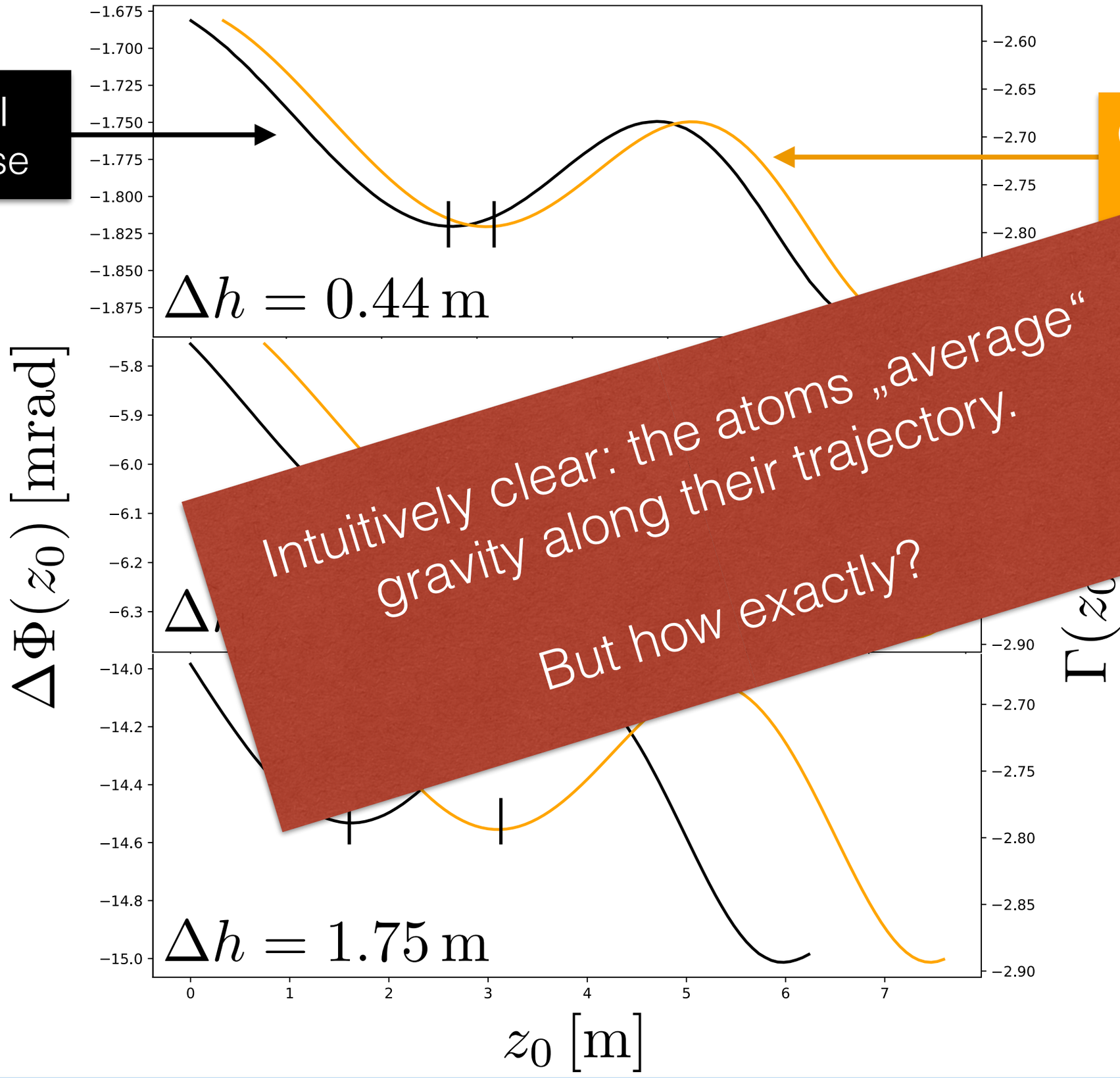
Example: Gravitational field of VLBAI Hannover [1, 2]



CGI: Numerical simulation in VLBAI

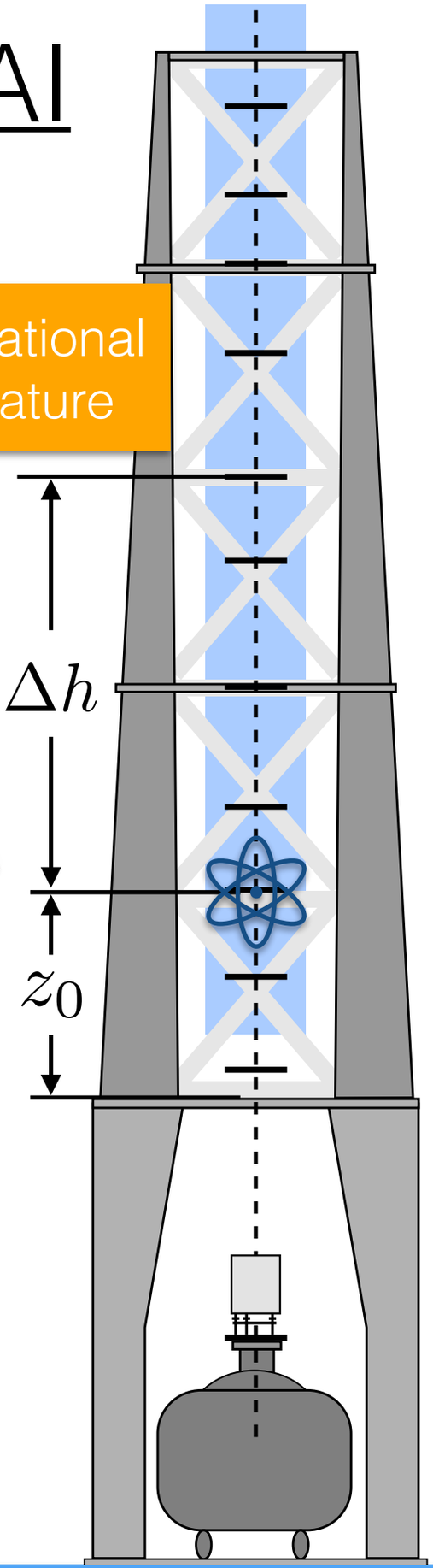
CGI Phase

Gravitational Curvature



Intuitively clear: the atoms „average“ gravity along their trajectory.

But how exactly?



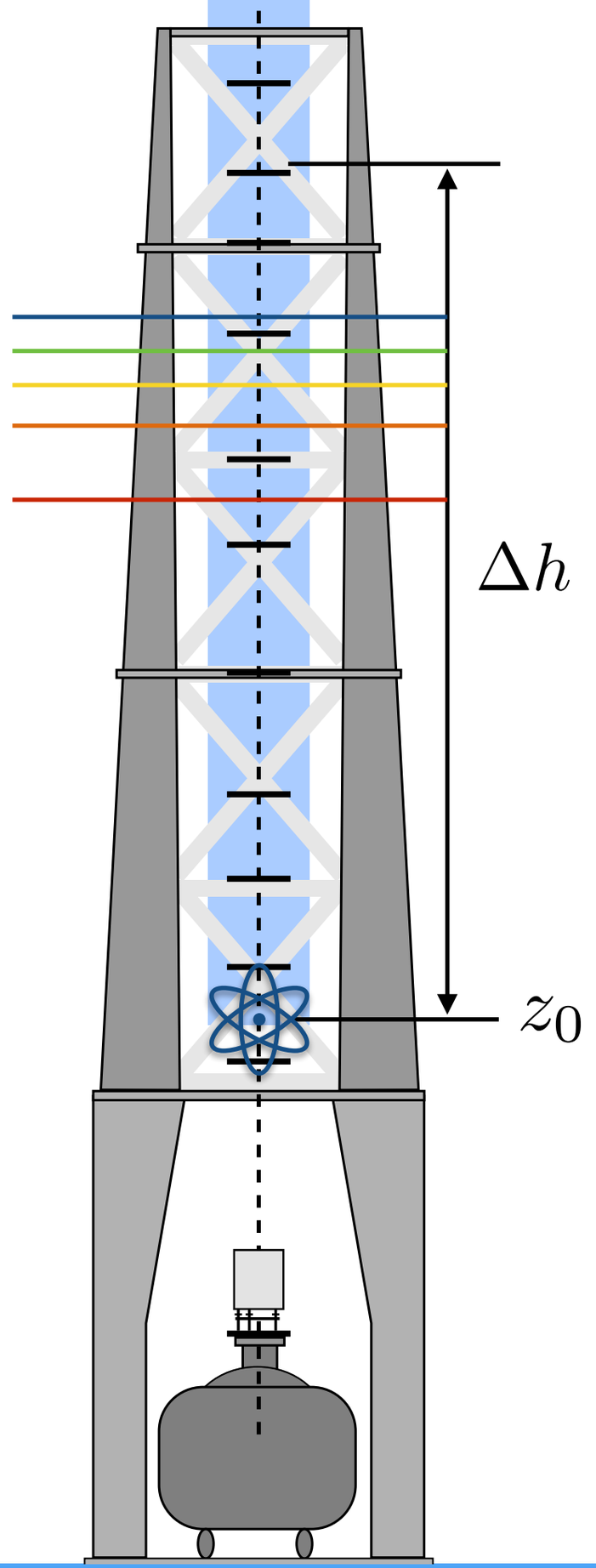
Averaging gravity

Where are the atoms „on average“?

Answer: It depends on the averaging process!

$$\|z(t)\|_n = \left(\frac{1}{2T_R} \int_0^{2T_R} |z(t) - z_0|^n \right)^{1/n}$$

n	1	2	3	4	5
$\|z(t)\|_n$	0.66 Δh	0.73 Δh	0.77 Δh	0.79 Δh	0.82 Δh



Estimator for grav. curvature

We can now define a novel estimator $\hat{\Gamma}(z_0)$ for the grav. curvature:

Step 1: Convert phase to grav. curvature using the scale factor.

$$\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0)}{f}$$

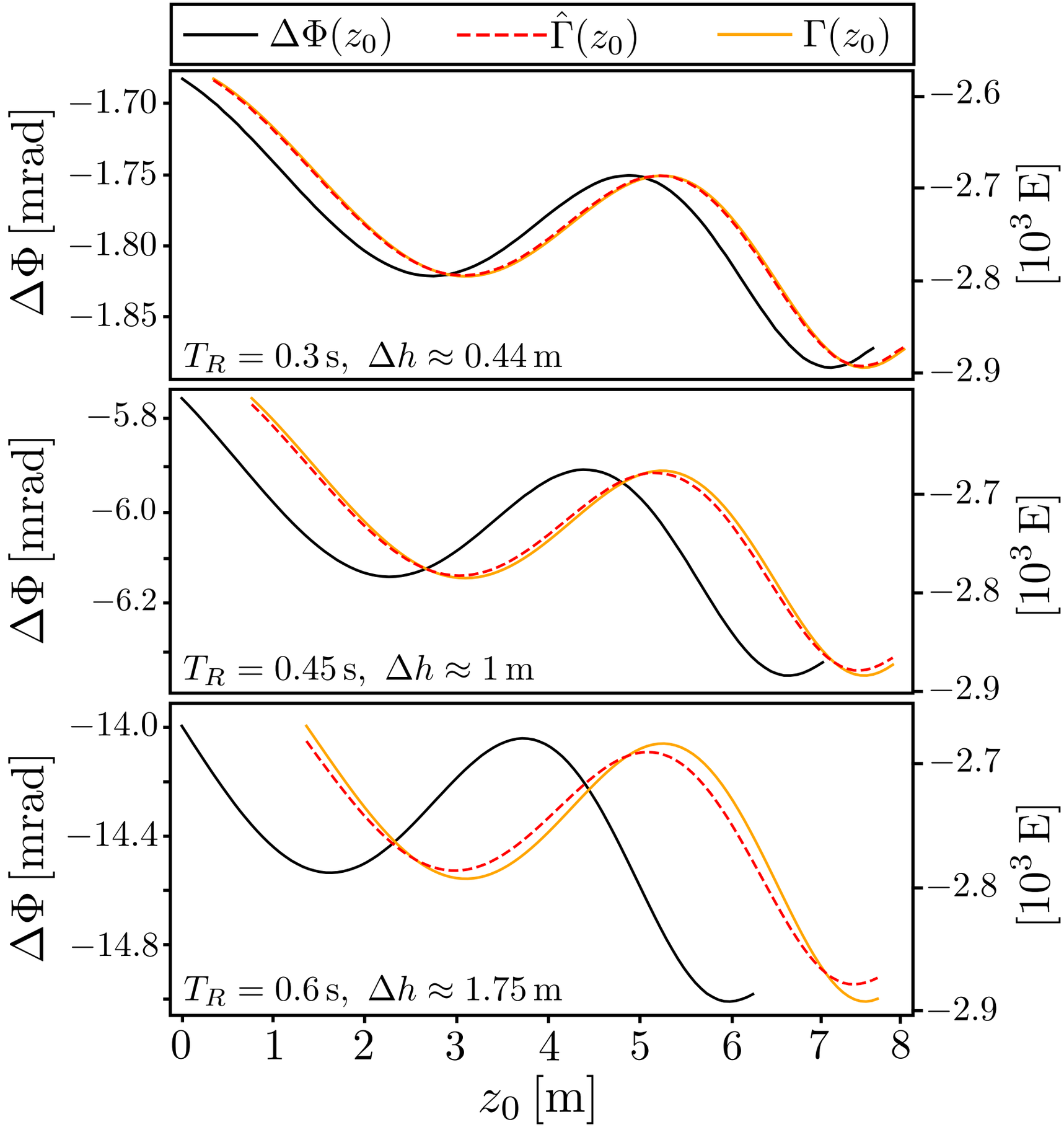
Step 2: Shift the height via the **cubic** mean of the trajectory.

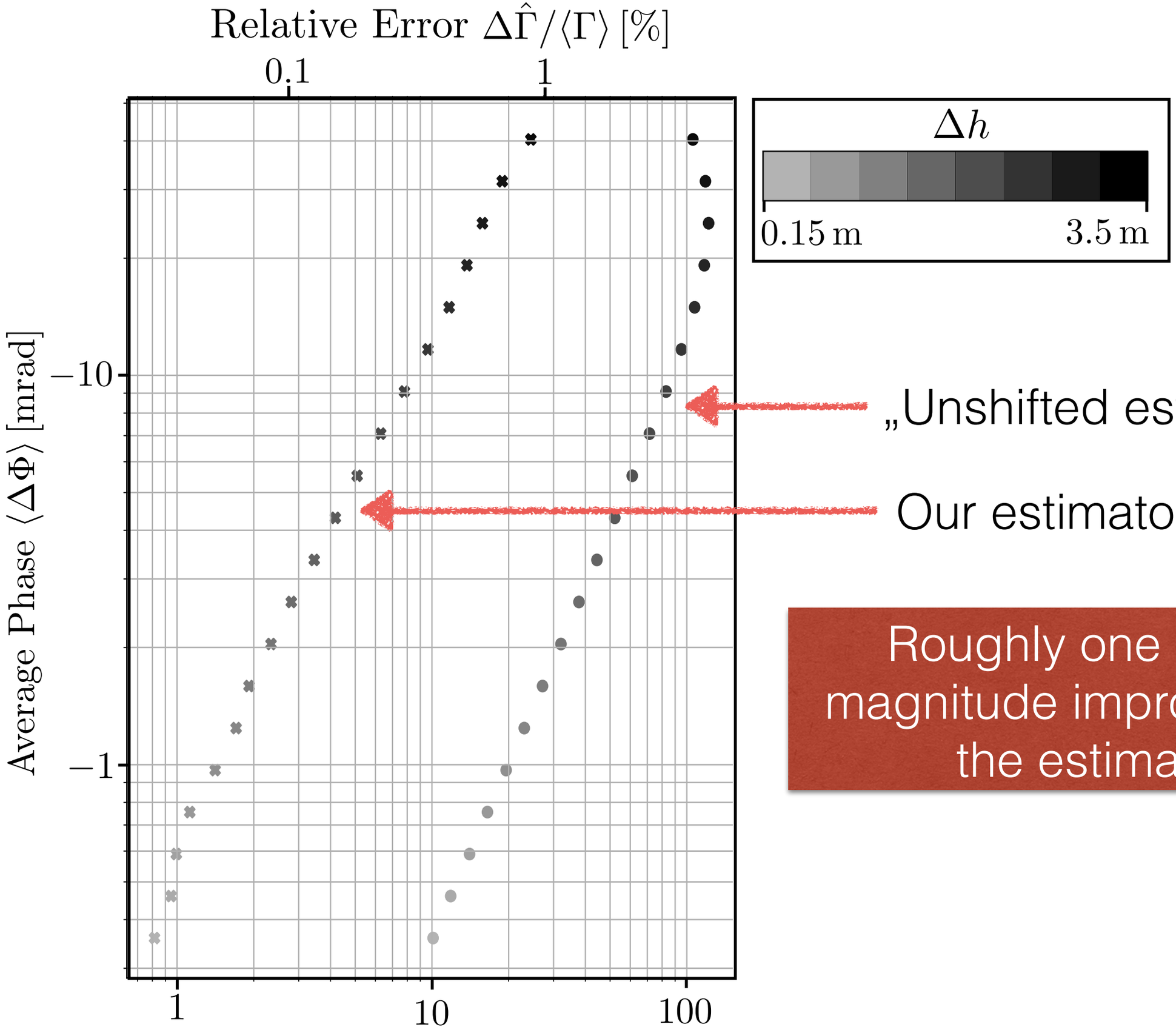
$$\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0 - \|z(t)\|_3)}{f}$$

Reminder:

$$f = -2\frac{N^2\hbar k_R^2 T_R^3}{m}$$

Motivation for the **cubic** mean.


$\Delta\Phi(z_0)$
$\hat{\Gamma}(z_0)$
$\Gamma(z_0)$
$\Delta\Phi$ [mrad]
$[10^3\,\mathrm{E}]$
$T_R = 0.3\,\mathrm{s},\ \Delta h \approx 0.44\,\mathrm{m}$
$T_R = 0.45\,\mathrm{s},\ \Delta h \approx 1\,\mathrm{m}$
$T_R = 0.6\,\mathrm{s},\ \Delta h \approx 1.75\,\mathrm{m}$
z_0 [m]



„Unshifted estimator“ $\frac{\Delta\Phi}{f}$

Our estimator $\hat{\Gamma}$

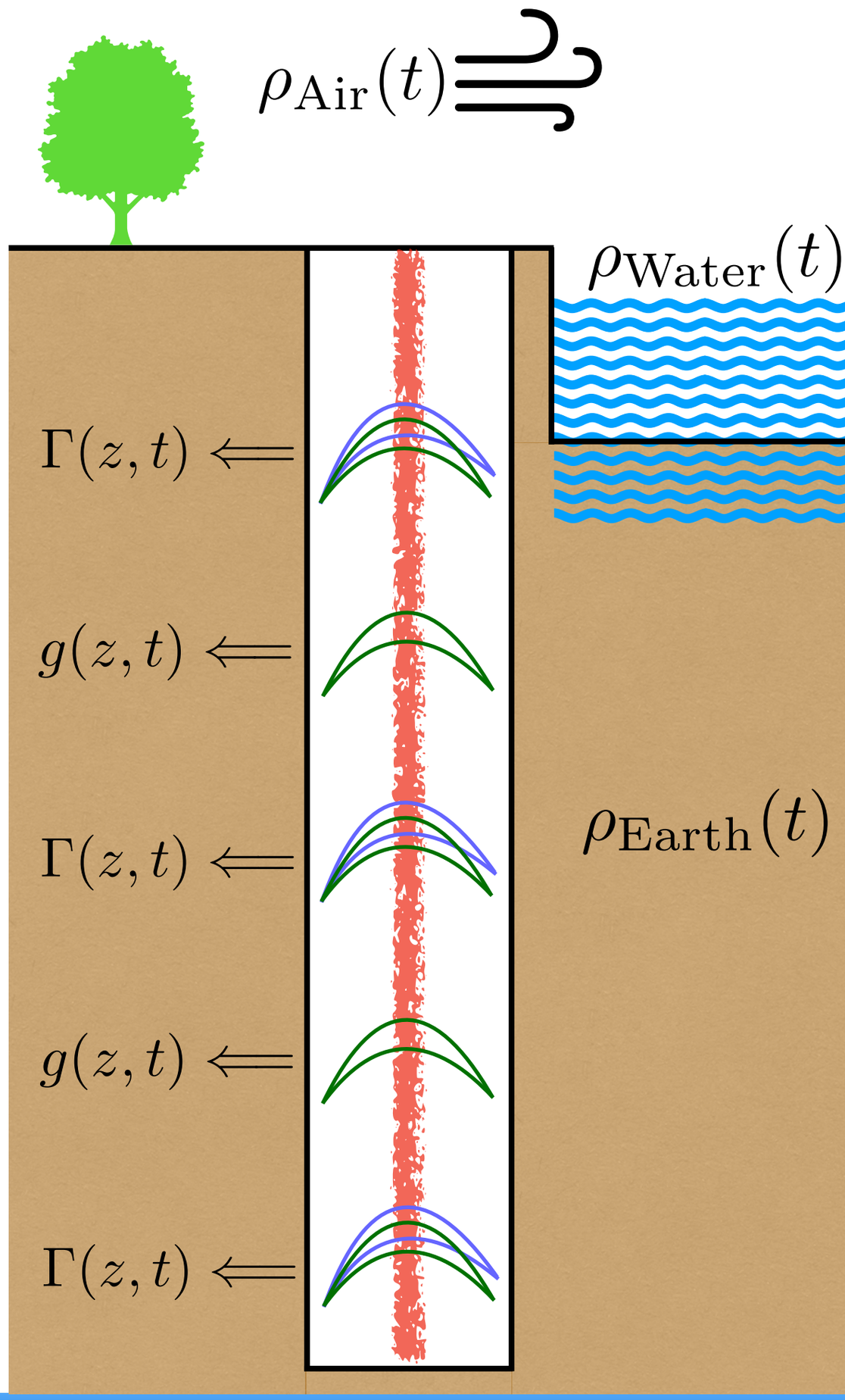
Roughly one order of magnitude improvement in the estimation!

Outlook / Summary

- We introduced the „CGI“ — giving us information about gravitational curvature.

- Gives rise to a novel way to measure gravitational field in large baseline interferometers.

- Possibility to measure the gravitational curvature using more compact devices.



Hammerer group

Thanks for listening!

Any questions?

Gaaloul group

Recent paper:

Link to the algorithm:

Other References:

- **[1]** Schilling et al. „Gravity field modelling for the Hannover 10m atom interferometer“

- **[2]** Lezeik et al. „Understanding the gravitational and magnetic environment of a very long baseline atom interferometer“

- **[3]** Schwartz et al. „Post-Newtonian Hamiltonian description of an atom in a weak gravitational field.“

Finite speed of light (FSL)

There are additional phase shifts, if one includes the FSL into the model!

Example: Two-Photon Bragg transitions would lead to an additional phase

$$\Delta\Phi_{\mathrm{FSL}} = \frac{4\hbar N^2 k^2 T_R}{mc}\left(4gT_R - v_0 - \frac{N\hbar k}{m}\right) + \Delta\Phi_0,$$

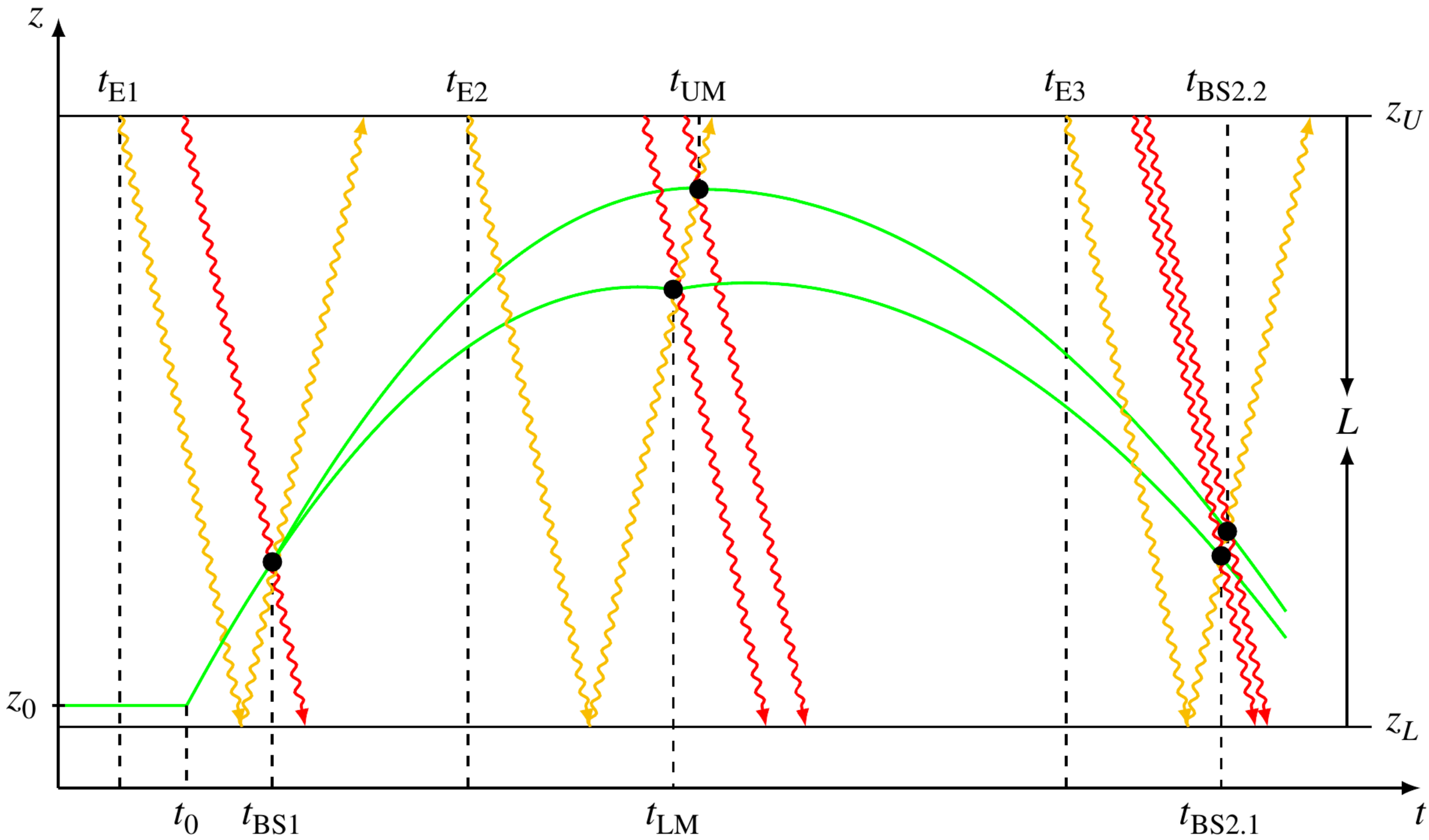
with a constant shift $\Delta\Phi_0 = \frac{2\hbar N^2 k^2}{mc}(2z_{\mathrm{L}} - z_0 - z_{\mathrm{U}}).$

Lower laser position

Upper laser position

Not that problematic, since constant in time.

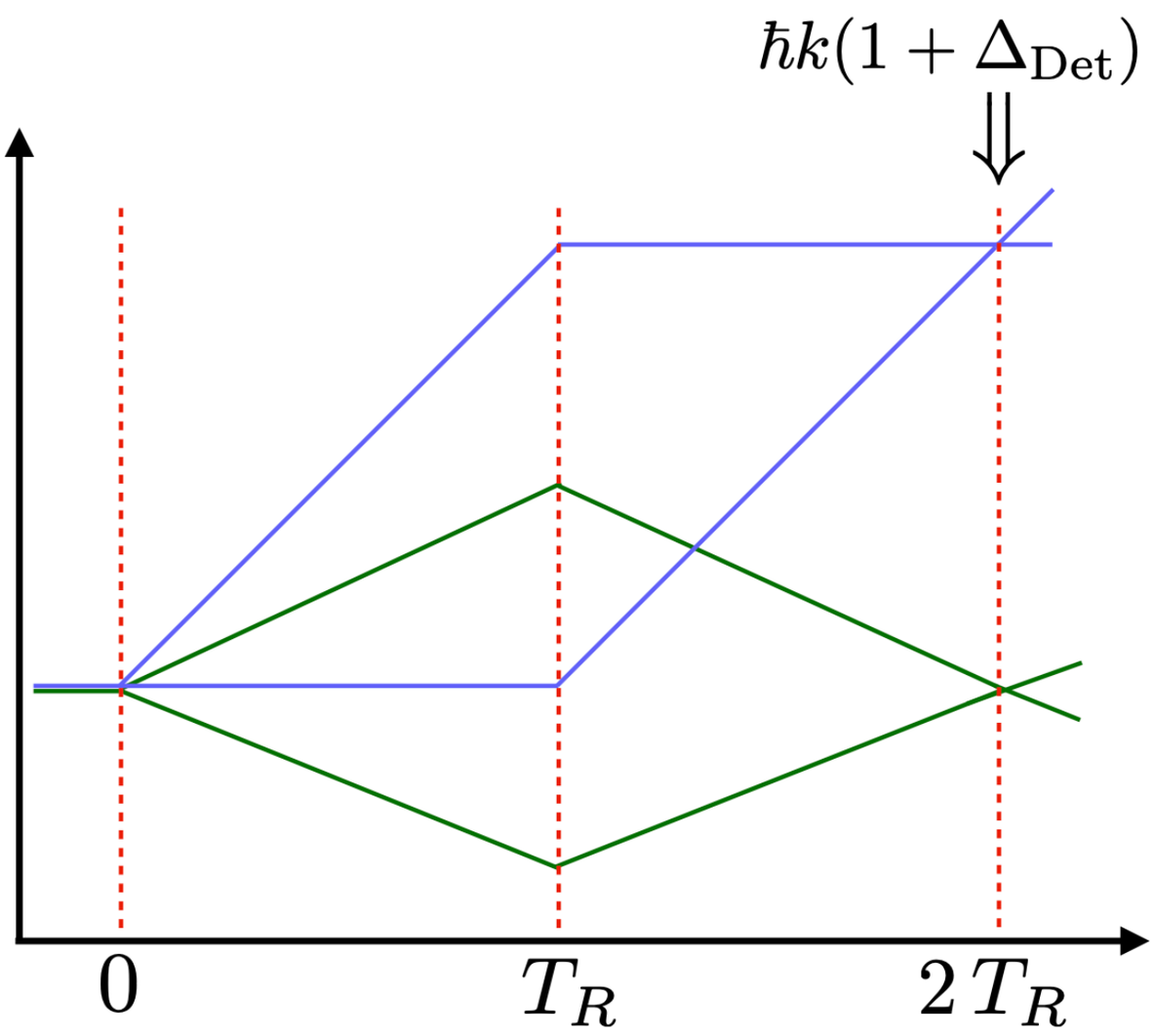
FSL with two-photon Bragg transitions



Photon paths of the example (FSL effect exaggerated).

FSL mitigation scheme

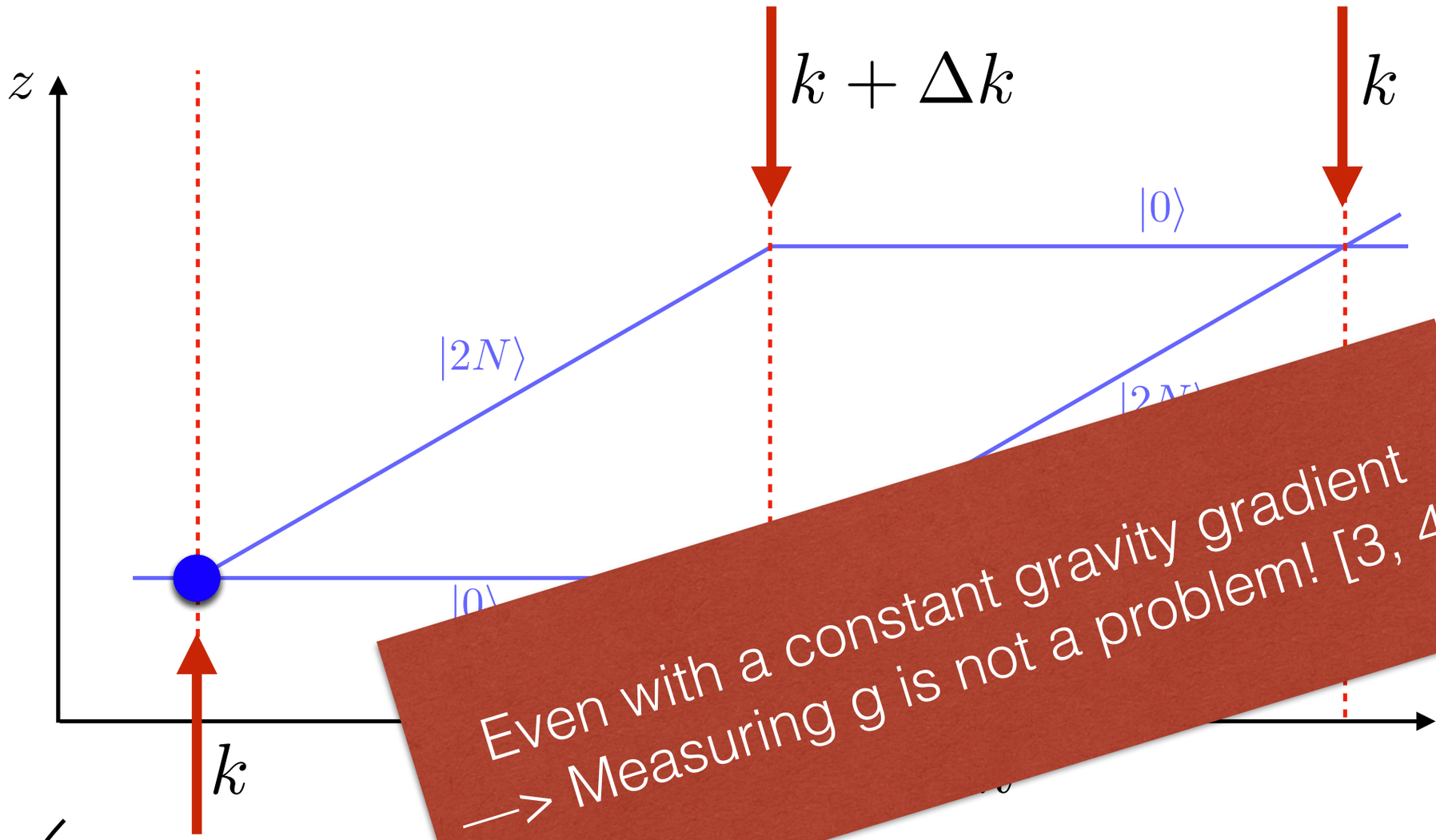
The time-dependent FSL phase can be mitigated!



Alter the last AIF pulse by:

$$\Delta_{\mathrm{Det}}(v_0, T_R) = 2\frac{v_0 + \frac{\hbar k}{m} - 4gT_R}{v_0 + \frac{\hbar k}{m} - gT_R}\frac{\hbar k}{mc}$$

This frequency chirp ($\sim 100\,\mathrm{MHz}$) nullifies the FSL phase.



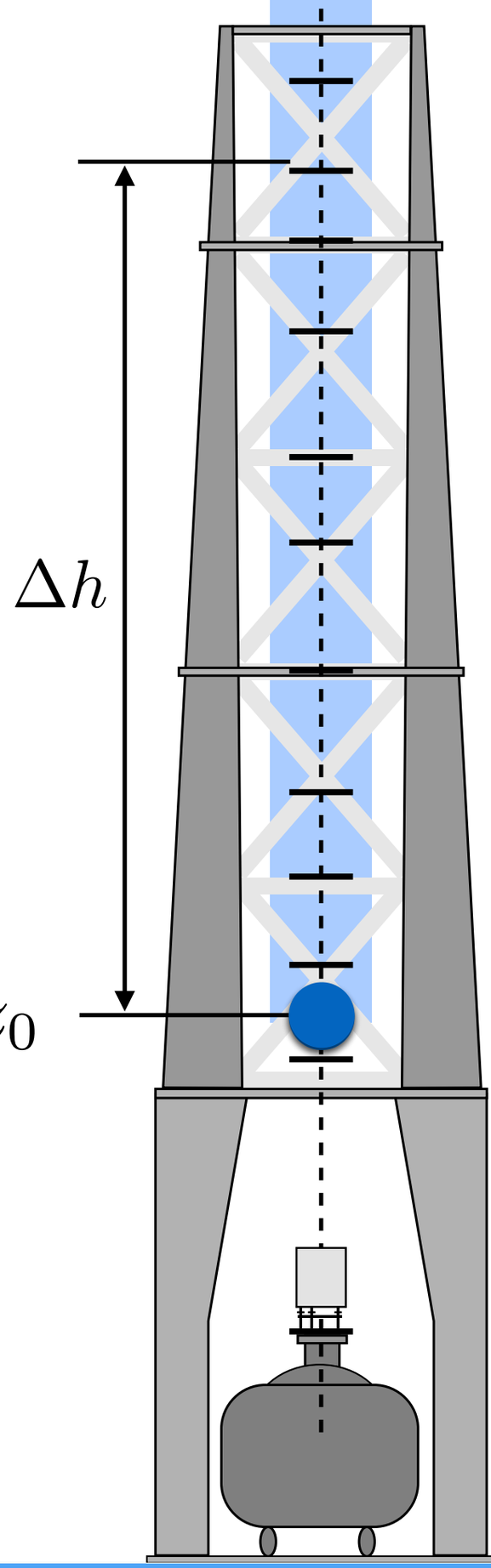
Even with a constant gravity gradient
—> Measuring g is not a problem! [3, 4]

Phase shift:

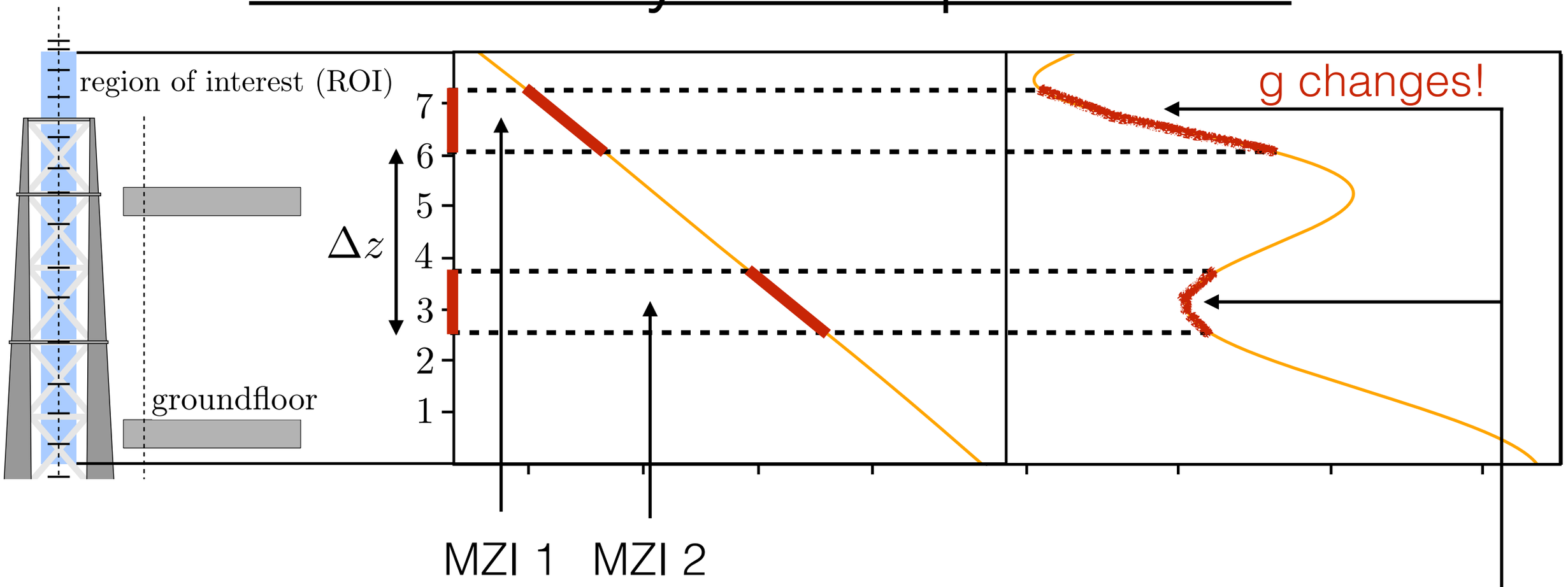
$$\Delta k = \frac{1}{2}\Gamma_0 T_R^2 k$$

$$\Delta\Phi = 2NgkT_R^2 + \Gamma_0 Nk\underbrace{\left(2z_0T_R^2 + 2v_0T_R^3 - \frac{7}{6}gT_R^4\right)} + \ldots$$

„Unwanted“ additional phases



Gradiometry in complex fields



$$\hat{\Gamma}_0 = \frac{\Delta g}{\Delta z}$$

Each Mach-Zehnder Interferometer (MZI) measures „its“ g.

Divide change in g over the height difference.

If you want high spatial resolution —> Make Δz and Δh small

—> Higher measurement uncertainty