Tidal Phase-Shifts in Atom Interferometry: Case Study of the VLBAI Hannover

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Introduction & Motivation

- Atom Interferometers become ever more accurate as quantum sensors, especially regarding measurements of gravity. We previously analysed how (general) relativistic effects alter interferometers (IFs) in idealised gravitational fields [1, 2].
- Additionally, local gravitational effects have been analysed in the context of „spacetime curvature“ and gravitational tidal effects [3]. Also in recent gravitational Aharonov-Bohm-type experiments [4], macroscopic test masses have been used to generate additional sources of the gravitational field.
- However, local gravitational effects like those can also be unintentional: Ground water variations, concrete structures, lab equipment, or even people disturb the gravitational environment. A detailed theoretical model of non-ideal gravitational fields becomes evermore important.

$\Longrightarrow$ Using the gravitational field measurements of the VLBAI in Hannover, we can model future experimental setups very accurately. We do so using an open source Python algorithm.

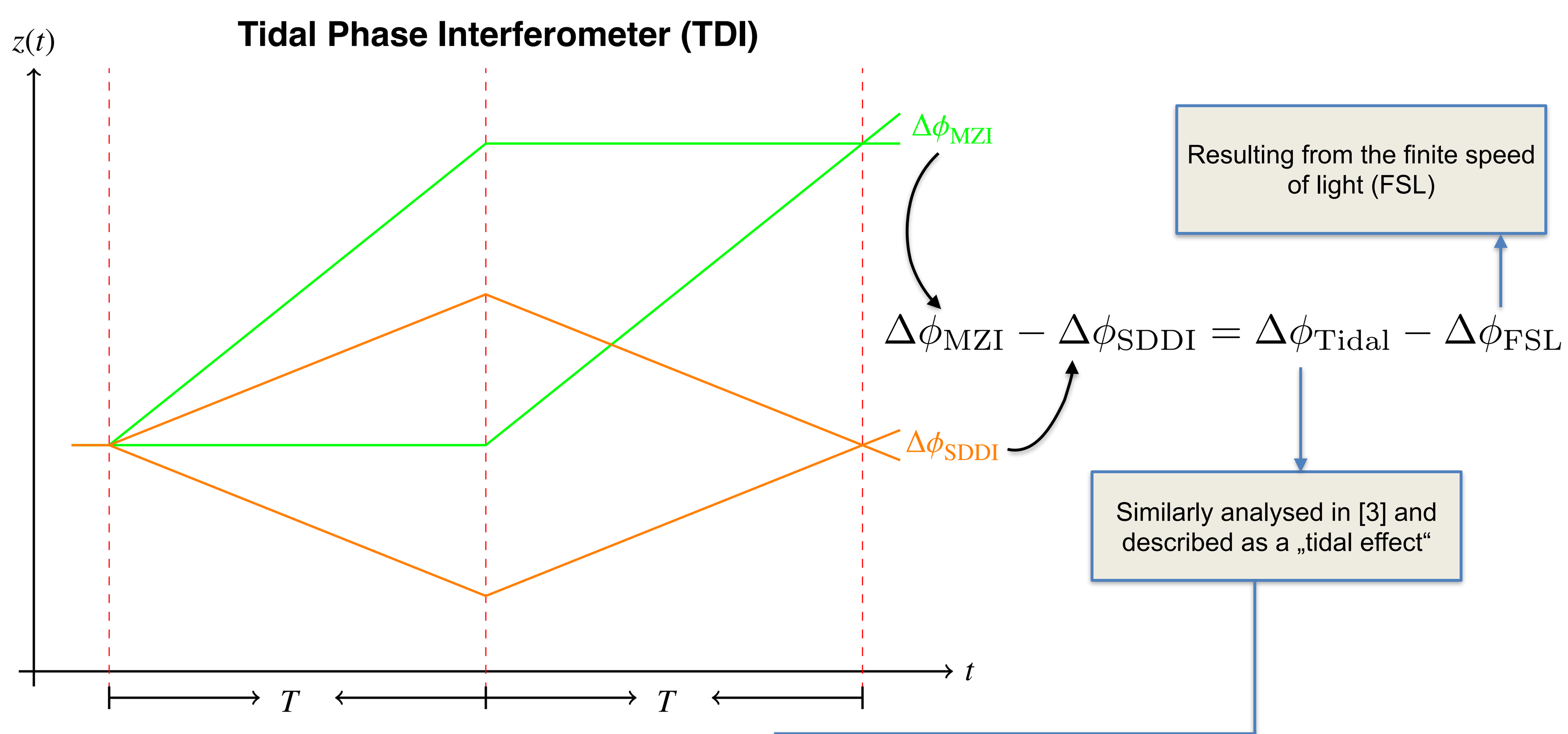
$\Longrightarrow$ We present a novel IF geometry that — dominantly — results in a phase which is connected to local gravitational field fluctuations.

Novel Interferometer Geometry for Tidal Phases

Gravitational potential expressed via its Taylor series around the origin:

$$\phi(z) = \sum_{n=0}^{\infty} \frac{\phi^{(n)}(0)}{n!} z^n$$

Consider the following differential interferometer geometry between a Mach-Zehnder Interferometer (MZI) and a Symmetric Double Diffraction Interferometer (SDDI):



$$\Delta\phi_{\mathrm{Tidal}} \approx \frac{m}{\hbar} \sum_{n=2}^{\infty} \frac{\phi^{(n)}(0)}{n!} \int_0^{2T} \left(z_{\mathrm{upper}}(t)^n - z_{\mathrm{lower}}(t)^n\right) \mathrm{d}t$$

This „tidal phase“ arises from the propagation phase along the quadratic term in the gravitational potential. It is therefore at least **cubic in time** and **quadratic in the photon recoil**.

The interpretation as a „tidal“ effect comes from this quadratic behaviour and is similar to the effect discussed in [3], i.e.

$$\Delta\phi_{\mathrm{Tidal}} = -\frac{\hbar N^2 k^2 T_{zz} T^3}{2m}$$ as in [3] with a grav. non-linearity $T_{zz} = \phi^{(2)}(0)$.

Mitigating the FSL phase shift:

The FSL phase depends on the explicit experimental setup and the type of laser interactions, i.e. being single, or two-photon interactions and the photon path lengths inside of the IF baseline [6].

Consider Bragg transitions, i.e. two light fields with individual wave vectors $k_1, -k_2$ and effective wave vector $k = k_1 + k_2$. The FSL phase is given by:

$$\Delta\phi_{\mathrm{FSL}} = \frac{4\hbar N^2 k^2 T}{mc}\left(4gT - v_0 - \frac{N\hbar k}{m}\right)$$

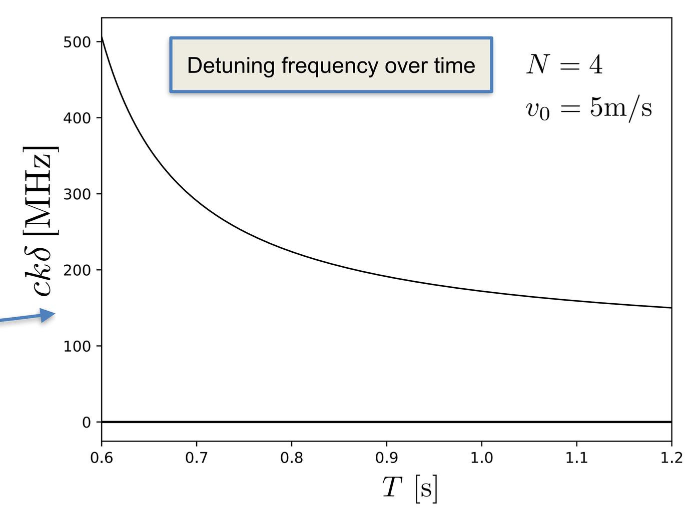
Introduce a detuning of the last pulse like $\hbar k \longmapsto (1+\delta)\,\hbar k$ with $\delta \ll 1$.

Aquire an additional phase of $\Delta\phi_{\mathrm{Additional}} = 2NkT\left(v_0 + \frac{N\hbar k}{m} - gT\right)\delta$

Choose detuning (as a function of v_0 and T) such that FSL phase cancels.

$$\delta(v_0, T) = \left(6\frac{v_0 + \frac{N\hbar k}{m}}{gT - v_0 - \frac{N\hbar k}{m}} + 8\right)\frac{N\hbar k}{mc}$$

Note: Denominator can get zero for some combinations of v_0 and T.



Phase Origin - Idealized Gravitational Field

To get an understanding of how this phase originates, let us consider the idealized gravitational potential of Earth, i.e.

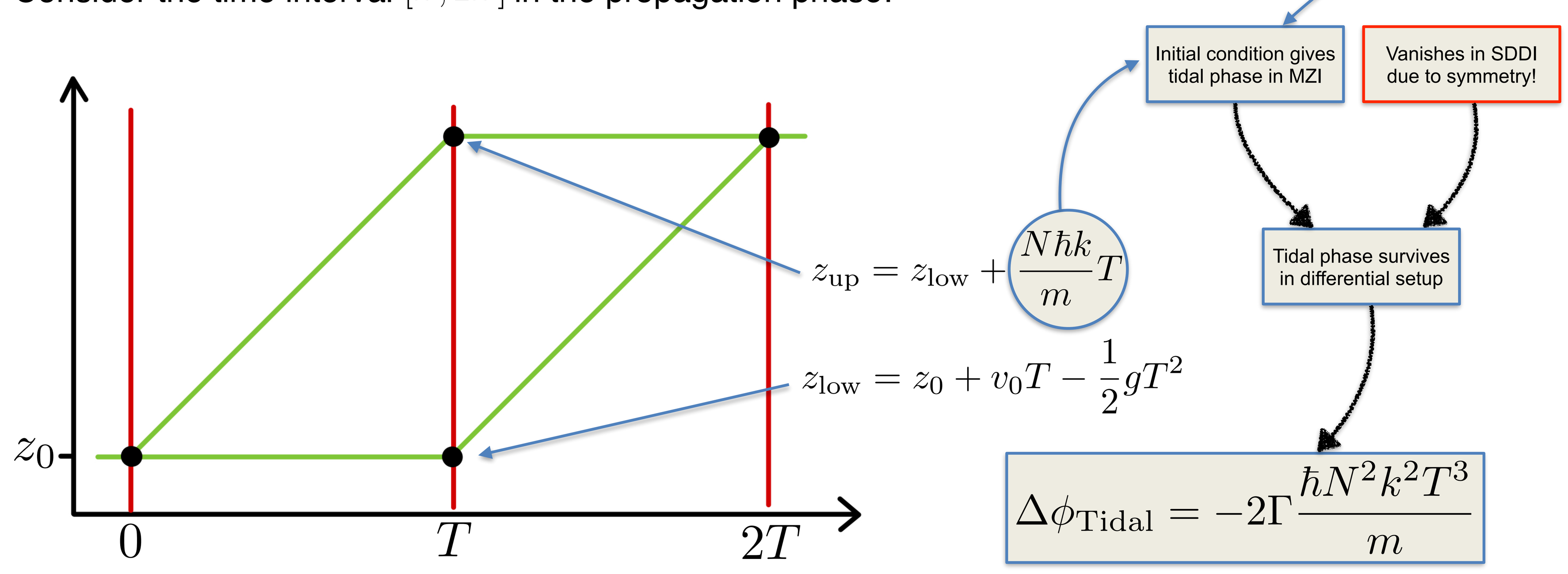
$$\phi(z) = gz - \frac{1}{2}\Gamma z^2 + \mathcal{O}(z^3)$$

The propagation phase is given by the action functional difference along the upper and lower path:

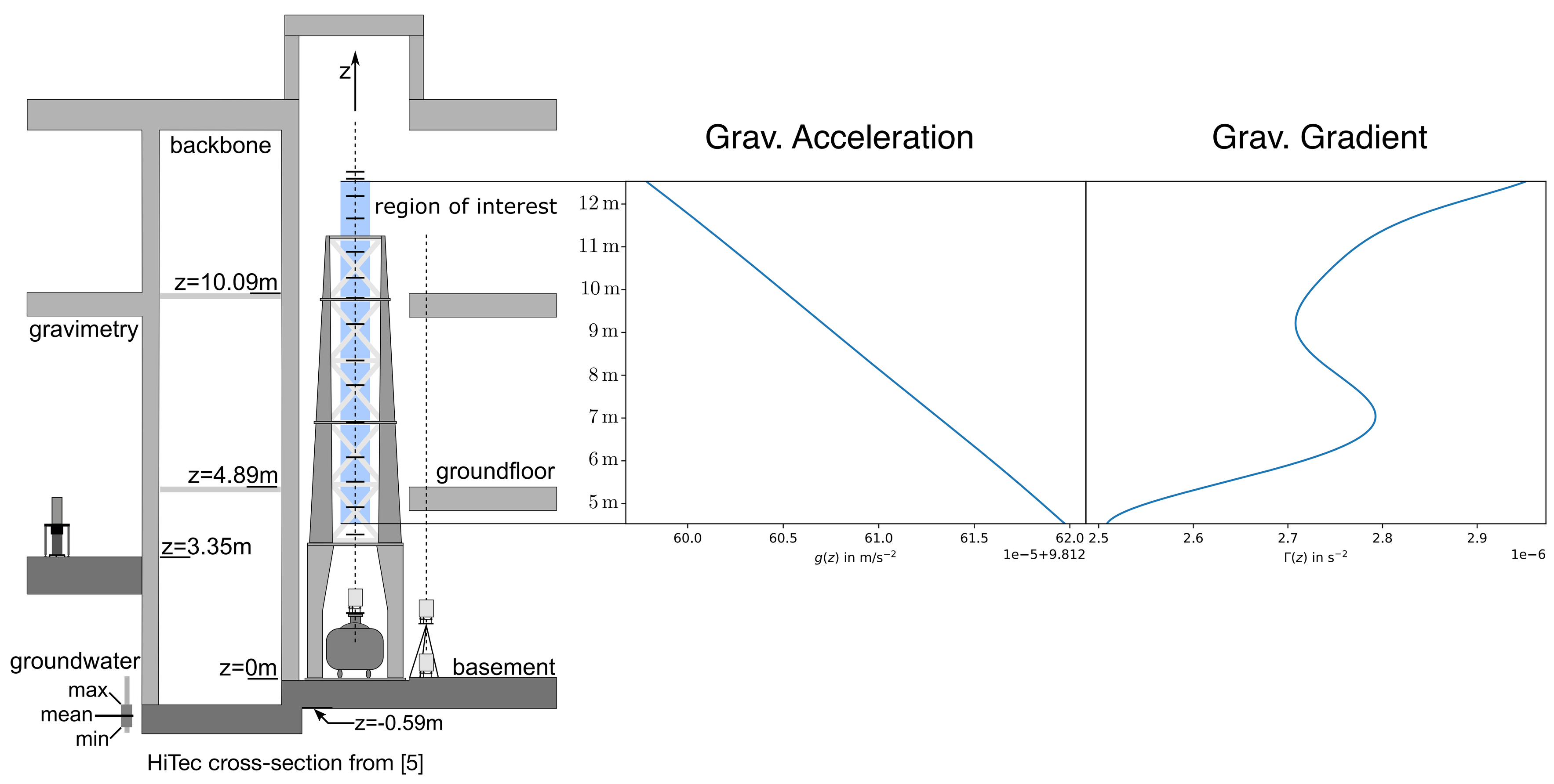
$$\Delta\phi_{\mathrm{Prop}} = \frac{1}{\hbar}\left(\int L(z_{\mathrm{upper}})\,\mathrm{d}t - \int L(z_{\mathrm{lower}})\,\mathrm{d}t\right)$$

With the Lagrangian corresponding to the COM Hamiltonian $L(z) = \frac{m}{2}\dot{z}^2 - mgz + \frac{m}{2}\Gamma z^2$

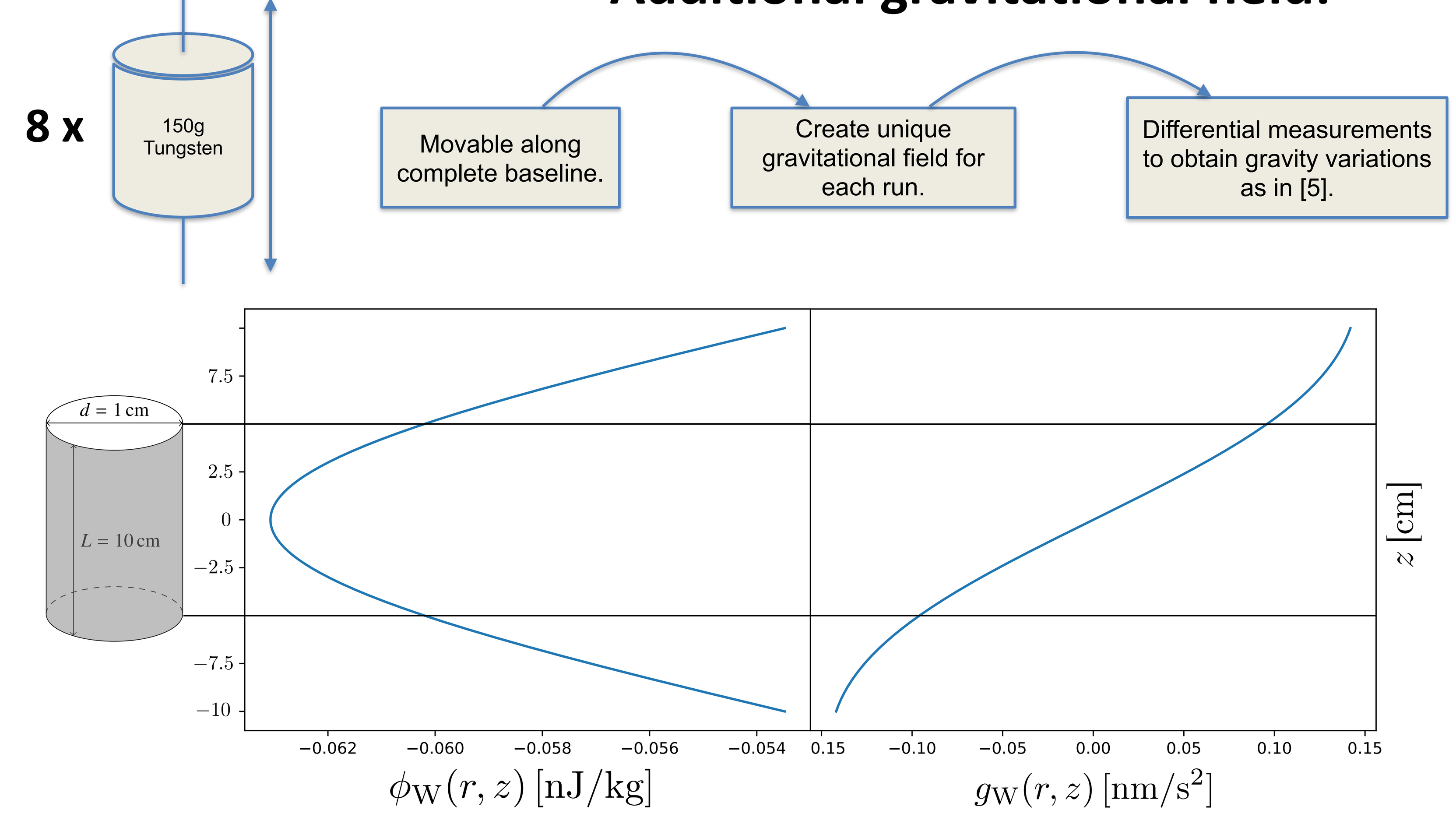
Consider the time interval $[T, 2T]$ in the propagation phase:



Real Gravitational Field: VLBAI Hannover



Additional gravitational field:



References

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Python code [2]