



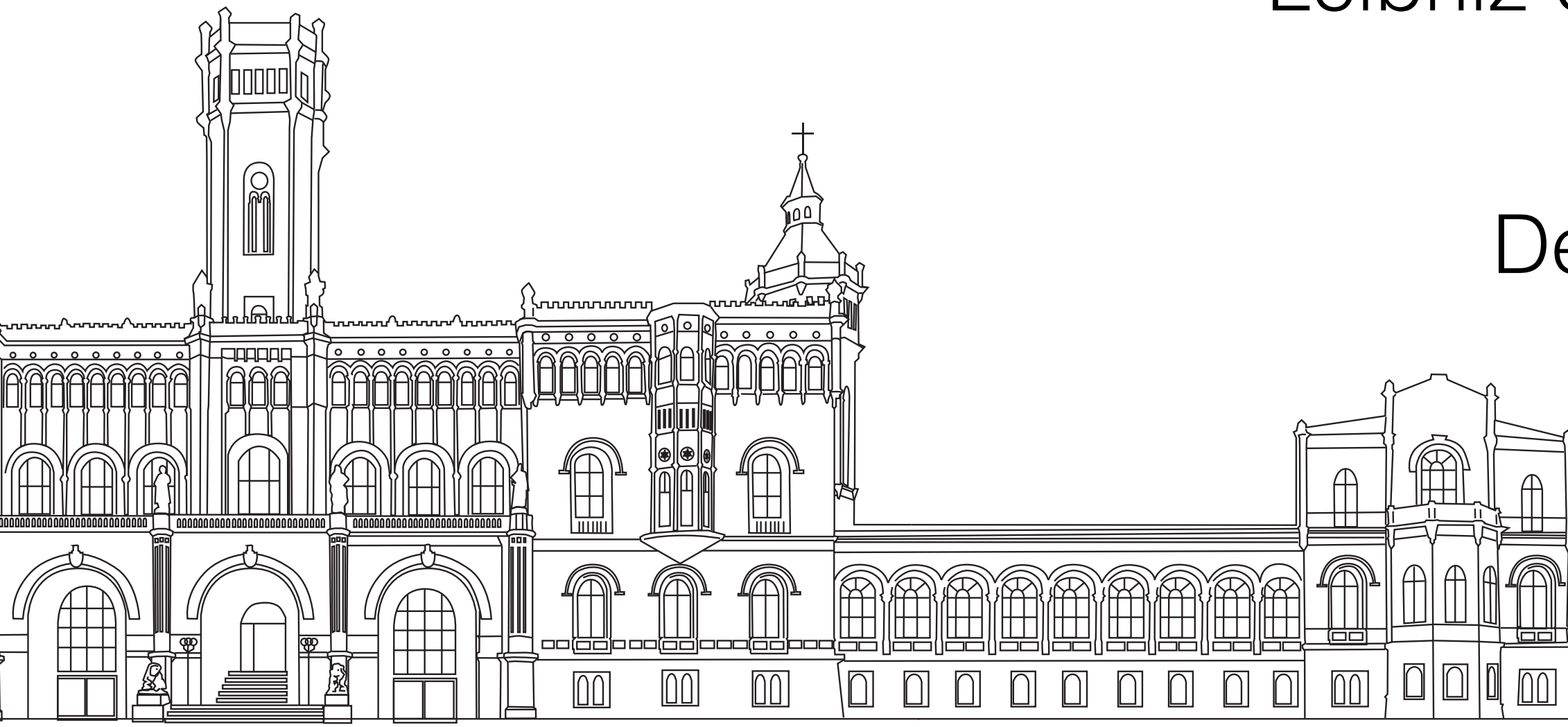
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Systematic analysis of atom interferometric measurements in complex gravitational environments

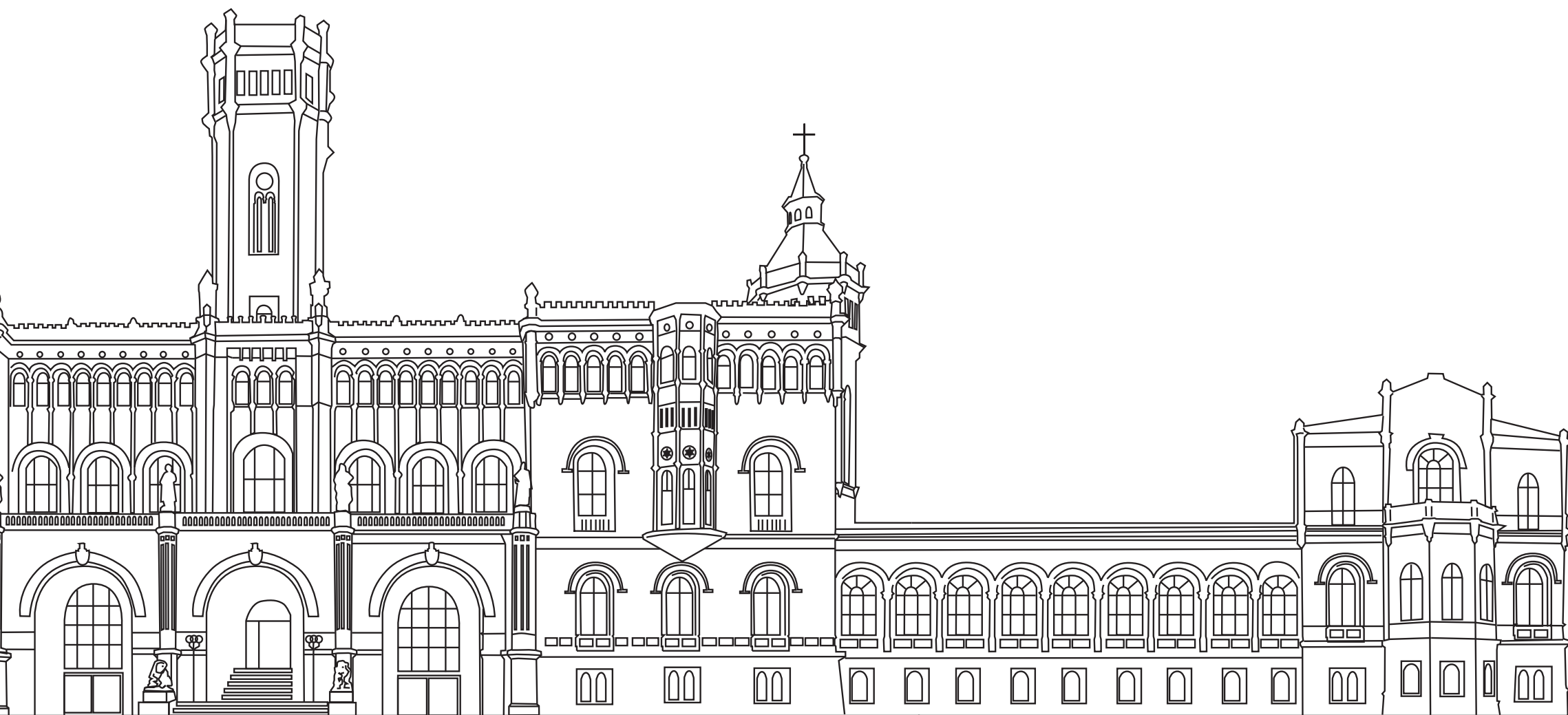
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Theory of Quantum Sensing
Leibniz University Hannover

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1. Motivation



How it started...

Theory colleagues working on general relativity (GR) developed this:

Post-Newtonian Hamiltonian description of an atom in a weak gravitational field

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We wanted to apply this novel GR-Hamiltonian to atom interferometers (AIFs)!

See Schwartz und Giulini, *Post-Newtonian Hamiltonian description of an atom* (2019)

How it started...

Minkowski space 

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

$\phi = \text{Gravitational potential}$

Newtonian spacetime  

$$g_{\mu\nu} = \begin{pmatrix} -1 - 2\frac{\phi}{c^2} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

(Parametrized) Post-Newtonian (PPN) spacetime   

$$g_{\mu\nu} = \begin{pmatrix} -1 - 2\frac{\phi}{c^2} - 2\beta\frac{\phi^2}{c^4} & 0 \\ 0 & \left(1 - 2\gamma\frac{\phi}{c^2}\right)\mathbb{1} \end{pmatrix}$$

PPN parameters

in GR: $\beta = \gamma = 1$

How it started...

$$H_{\text{C,final}} = \frac{\mathbf{P}^2}{2M} \left[1 - \frac{1}{Mc^2} \left(\frac{\mathbf{p}_r^2}{2\mu} + \frac{e_1 e_2}{4\pi\epsilon_0 r} \right) \right] + \left[M + \frac{1}{c^2} \left(\frac{\mathbf{p}_r^2}{2\mu} + \frac{e_1 e_2}{4\pi\epsilon_0 r} \right) \right] \phi(\mathbf{R}) \\ - \frac{\mathbf{P}^4}{8M^3 c^2} + \frac{2\gamma + 1}{2Mc^2} \mathbf{P} \cdot \phi(\mathbf{R}) \mathbf{P} + (2\beta - 1) \frac{M \phi(\mathbf{R})^2}{2c^2} ,$$

Center of mass
(COM)

Look @ elastic scattering processes (first)
—> Populate the excited state only virtually.

Internal dynamics

$$H_{\text{AL,final}} = -\mathbf{d} \cdot \mathbf{E}_{\text{coord.}}^{\perp}(\mathbf{R}) + \frac{1}{2M} \{ \mathbf{P} \cdot [\mathbf{d} \times \mathbf{B}_{\text{coord.}}(\mathbf{R})] + \text{H.c.} \} \\ - \frac{m_1 - m_2}{4m_1 m_2} \{ \mathbf{p}_r \cdot [\mathbf{d} \times \mathbf{B}_{\text{coord.}}(\mathbf{R})] + \text{H.c.} \} \\ + \frac{1}{8\mu} (\mathbf{d} \times \mathbf{B}_{\text{coord.}}(\mathbf{R}))^2 + \frac{1}{2\epsilon_0} \int d^3 \mathbf{x} \left(1 + (\gamma + 1) \frac{\phi}{c^2} \right) \mathcal{P}_d^{\perp 2}(\mathbf{x}, t) ,$$

Atom-Light-
Interaction

Treat light fields classical
—> Maxwell's equations

Light field

COM Hamiltonian: Propagation Phase

We follow a semiclassical approach with

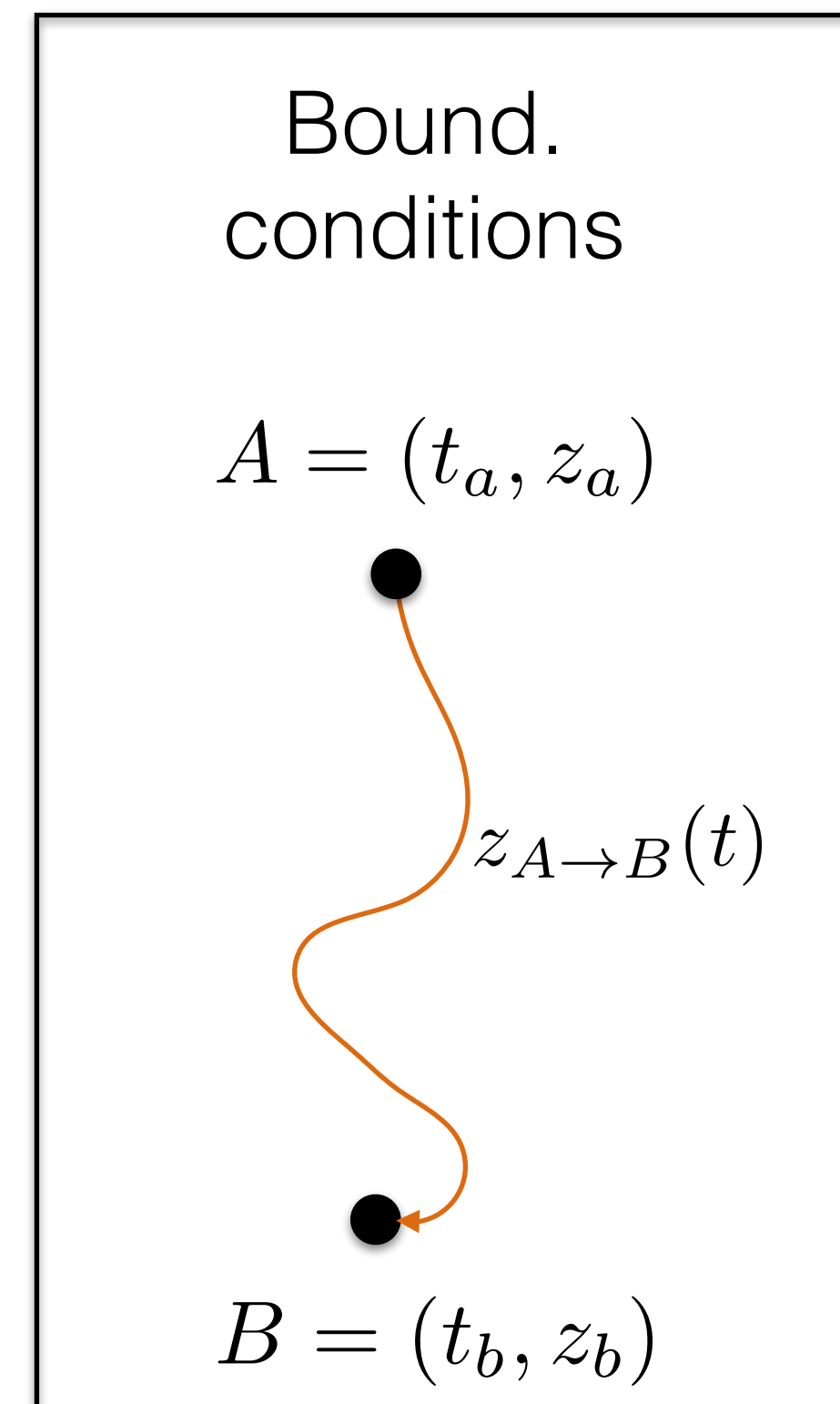
$$L(z, \dot{z}) = \boxed{-mc^2}_{\text{🌶️}} + \boxed{\frac{m\dot{z}^2}{2} - m\phi(z)}_{\text{🌶️🌶️}} + \boxed{\frac{1}{mc^2} \left[\frac{m^2 \dot{z}^4}{8} - \frac{2\beta - 1}{2} m^2 \phi(z)^2 - \frac{2\gamma + 1}{2} m\phi(z) \dot{z}^2 \right]}_{\text{🌶️🌶️🌶️}}$$

The phase of an atom then evolves freely as

$$\Delta\Phi_{A \rightarrow B} = \frac{1}{\hbar} \int_{t_a}^{t_b} dt L(z_{A \rightarrow B}(t), \dot{z}_{A \rightarrow B}(t))$$

where the atomic trajectory solves the Euler-Lagrange equation

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} = 0$$



Light Hamiltonian: Maxwell's equations

We analyse Maxwell's equations in vacuum:

$$F_{\alpha\beta}{}^{;\beta} = \nabla^\beta F_{\alpha\beta} = \nabla^\beta (\nabla_\alpha A_\beta - \nabla_\beta A_\alpha) = 0$$

Three different choices for gauges:

$$\nabla_\beta A^\beta = A^\beta{}_{;\beta} = A_\beta{}^{;\beta} = 0,$$

Lorenz gauge

$$\nabla^i A_i = A_i{}^{;i} = 0,$$

Geometric Coulomb gauge

$$\partial^i A_i = A_i{}^{,i} = 0.$$

Background Coulomb gauge

Maxwell's equations: Geometric Coulomb

Writing them out and using the **gauge condition**:

$$\begin{aligned}
 0 &= F_{\nu\mu}{}^{;\mu} = A_{\mu}{}^{;\mu}{}_{;\nu} - A_{\nu;\mu}{}^{;\mu} \\
 &= g^{\mu\lambda} \partial_{\lambda} \partial_{\mu} A_{\nu} - \partial_{\nu} \partial^{\mu} A_{\mu} - g^{\mu\lambda} \Gamma^{\sigma}{}_{\lambda\mu} (\partial_{\sigma} A_{\nu} + \partial_{\nu} A_{\sigma}) - g^{\mu\lambda} \Gamma^{\sigma}{}_{\lambda\nu} \partial_{\mu} A_{\sigma} \\
 &= g^{\mu\lambda} \partial_{\lambda} \partial_{\mu} A_{\nu} - \partial_{\nu} \partial^0 A_0 - \partial_{\nu} \partial^i A_i - g^{\mu\lambda} \Gamma^{\sigma}{}_{\lambda\mu} \partial_{\sigma} A_{\nu} + \partial_{\nu} g^{0\lambda} \Gamma^{\sigma}{}_{\lambda 0} A_{\sigma} \\
 &\quad - (\partial_{\nu} g^{\mu\lambda}) \Gamma^{\sigma}{}_{\lambda\mu} A_{\sigma} - g^{\mu\lambda} (\partial_{\nu} \Gamma^{\sigma}{}_{\lambda\mu}) A_{\sigma} - g^{\mu\lambda} \Gamma^{\sigma}{}_{\lambda\nu} \partial_{\mu} A_{\sigma} + g^{\mu\lambda} \Gamma^{\sigma}{}_{\lambda\nu} \partial_{\sigma} A_{\mu} \\
 &= g^{\mu\lambda} (\partial_{\lambda} \partial_{\mu} A_{\nu} - \Gamma^{\sigma}{}_{\lambda\mu} \partial_{\sigma} A_{\nu} - (\partial_{\nu} \Gamma^{\sigma}{}_{\lambda\mu}) A_{\sigma} - \Gamma^{\sigma}{}_{\lambda\nu} \partial_{\mu} A_{\sigma} + \Gamma^{\sigma}{}_{\lambda\nu} \partial_{\sigma} A_{\mu}) \\
 &\quad - \partial_{\nu} (\partial^0 A_0 - g^{0\lambda} \Gamma^{\sigma}{}_{\lambda 0} A_{\sigma}) - (\partial_{\nu} g^{\mu\lambda}) \Gamma^{\sigma}{}_{\lambda\mu} A_{\sigma} - \partial_{\nu} (\partial^i A_i - g^{ij} \Gamma^{\sigma}{}_{ij} A_{\sigma}) \\
 &= g^{00} (\partial_0^2 A_{\nu} - \Gamma^{\sigma}{}_{00} \partial_{\sigma} A_{\nu} - (\partial_{\nu} \Gamma^{\sigma}{}_{00}) A_{\sigma} - \Gamma^{\sigma}{}_{0\nu} \partial_0 A_{\sigma} + \Gamma^{\sigma}{}_{0\nu} \partial_{\sigma} A_0) \\
 &\quad - \partial_{\nu} (\partial^0 A_0 - g^{00} \Gamma^{\sigma}{}_{00} A_{\sigma}) - (\partial_{\nu} g^{00}) \Gamma^{\sigma}{}_{00} A_{\sigma} - (\partial_{\nu} g^{ij}) \Gamma^{\sigma}{}_{ij} A_{\sigma} \\
 &\quad + g^{ij} (\partial_i \partial_j A_{\nu} - \Gamma^{\sigma}{}_{ij} \partial_{\sigma} A_{\nu} - (\partial_{\nu} \Gamma^{\sigma}{}_{ij}) A_{\sigma} - \Gamma^{\sigma}{}_{j\nu} \partial_i A_{\sigma} + \Gamma^{\sigma}{}_{j\nu} \partial_{\sigma} A_i)
 \end{aligned}$$



Maxwell's equations: Geometric optics approximation

Ansatz with $\epsilon \ll 1$ of the following form:

$$A_\mu = (a_\mu + \epsilon b_\mu + \epsilon^2 c_\mu + \mathcal{O}(\epsilon^3)) e^{i\Phi/\epsilon}$$

Amplitudes

$$a_\mu, b_\mu, c_\mu : \mathcal{M} \longrightarrow \mathbb{C}$$

Phase

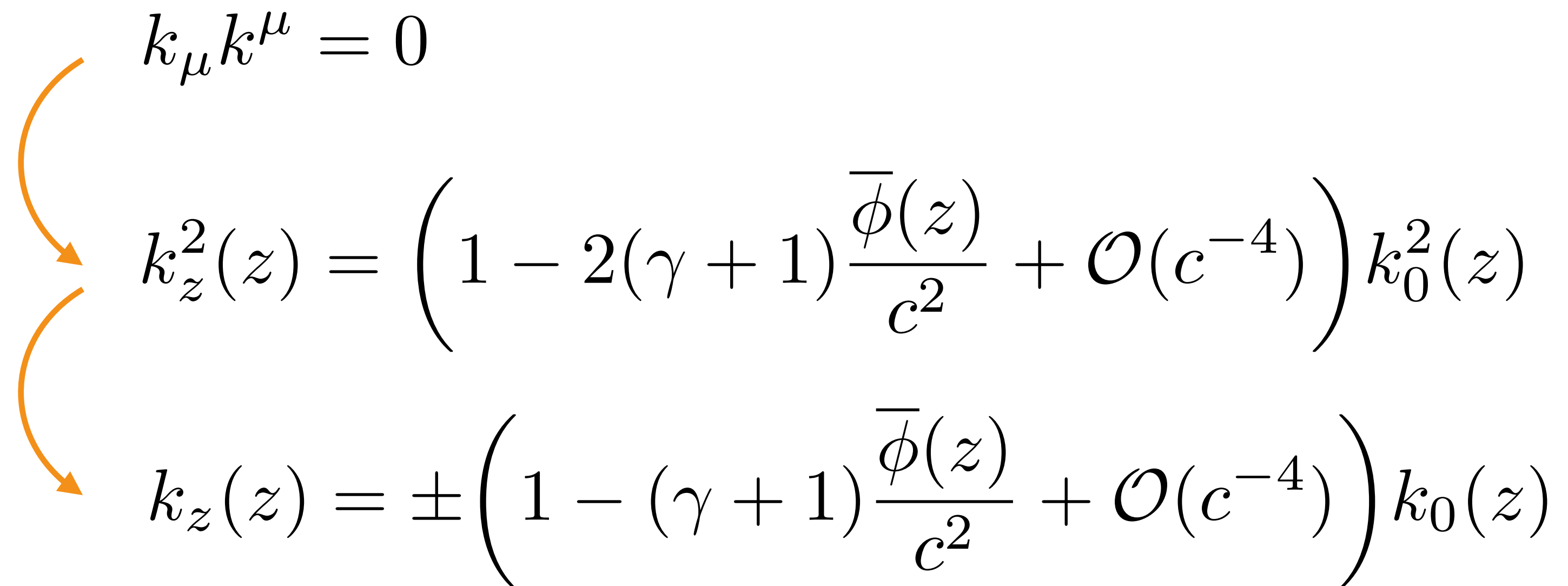
$$\Phi : \mathcal{M} \longrightarrow \mathbb{R}$$

$$k_\mu = \nabla_\mu \Phi = \partial_\mu \Phi$$

Maxwell's equations: Leading order

$$\mathcal{O}(\epsilon^{-2}) : k_\mu k^\mu a_\nu = 0$$

To leading order, geometric optics ensures that the wave vector is **light like**.



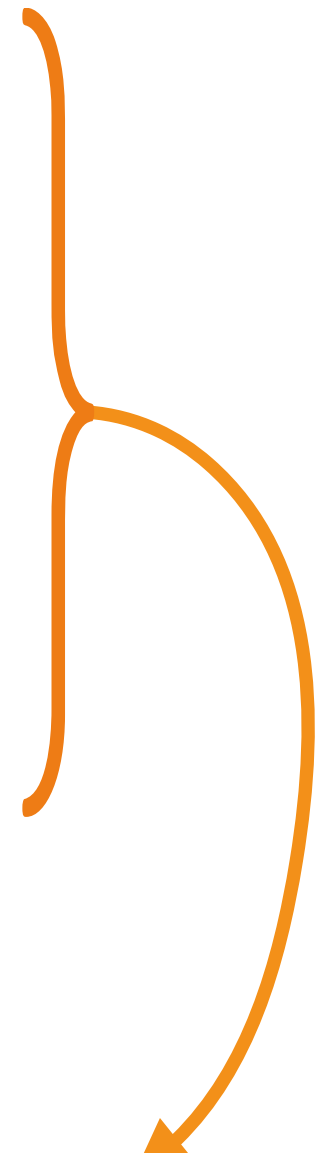
The diagram illustrates the derivation of the wave vector components from the light-like condition $k_\mu k^\mu = 0$. An orange curved arrow points from the equation $k_\mu k^\mu = 0$ to the equation $k_z^2(z) = \left(1 - 2(\gamma + 1)\frac{\bar{\phi}(z)}{c^2} + \mathcal{O}(c^{-4})\right)k_0^2(z)$. Another orange curved arrow points from the same equation $k_\mu k^\mu = 0$ to the equation $k_z(z) = \pm \left(1 - (\gamma + 1)\frac{\bar{\phi}(z)}{c^2} + \mathcal{O}(c^{-4})\right)k_0(z)$.

$$k_\mu k^\mu = 0$$
$$k_z^2(z) = \left(1 - 2(\gamma + 1)\frac{\bar{\phi}(z)}{c^2} + \mathcal{O}(c^{-4})\right)k_0^2(z)$$
$$k_z(z) = \pm \left(1 - (\gamma + 1)\frac{\bar{\phi}(z)}{c^2} + \mathcal{O}(c^{-4})\right)k_0(z)$$

Using some more algebra we also find that $k_0(z) = k_0$

Maxwell's equations: Sub-Leading order

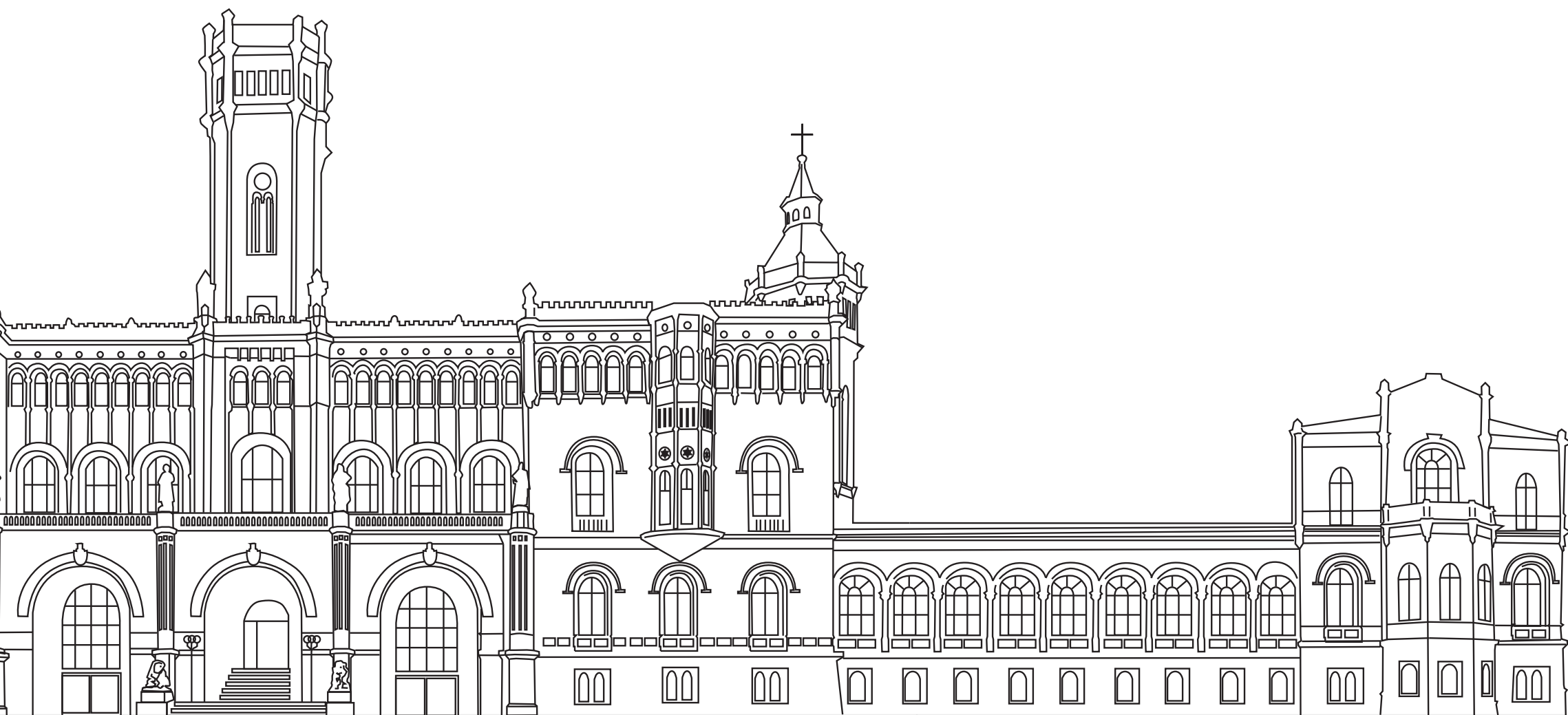
Next order equations: $\mathcal{O}(\epsilon^{-1})$

$$\begin{aligned}(\nabla a_x) \cdot \mathbf{k} + \frac{1}{2} a_x \nabla \cdot \mathbf{k} &= \frac{2\gamma + 1}{2c^2} (\partial_z \bar{\phi}) k_x a_z - \frac{\gamma + 1}{2c^2} (\partial_z \bar{\phi}) k_z a_x + \mathcal{O}(\epsilon^0, c^{-4}), \\(\nabla a_y) \cdot \mathbf{k} + \frac{1}{2} a_y \nabla \cdot \mathbf{k} &= \frac{2\gamma + 1}{2c^2} (\partial_z \bar{\phi}) k_y a_z - \frac{\gamma + 1}{2c^2} (\partial_z \bar{\phi}) k_z a_y + \mathcal{O}(\epsilon^0, c^{-4}), \\(\nabla a_z) \cdot \mathbf{k} + \frac{1}{2} a_z \nabla \cdot \mathbf{k} &= \frac{\gamma}{2c^2} (\partial_z \bar{\phi}) k_z a_z + \mathcal{O}(\epsilon^0, c^{-4}).\end{aligned}$$


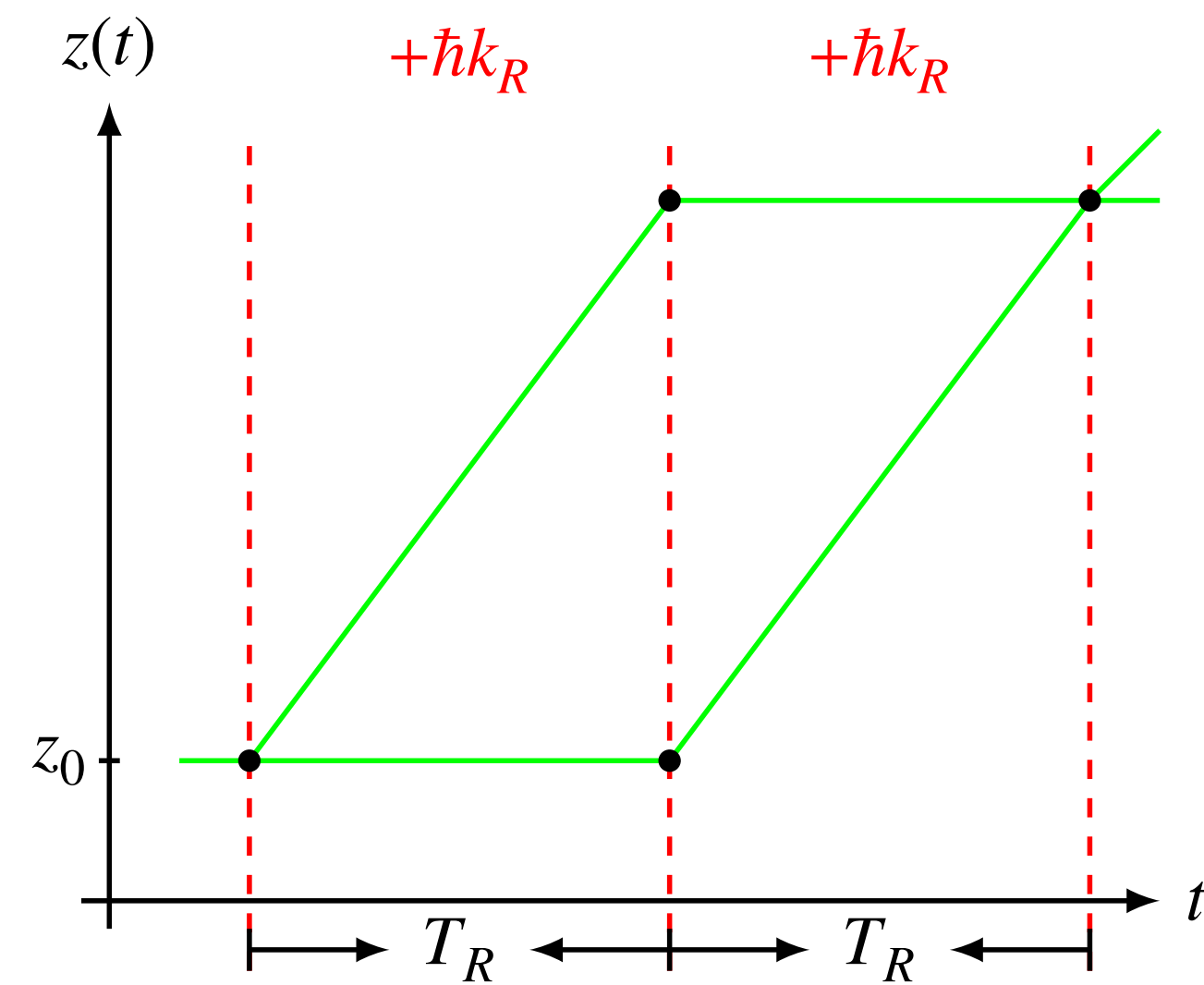
No corrections to the amplitudes to relevant order.

$$(A_i) = \mathbf{A} = \mathcal{A} e^{i\left(k_0 c t \pm \left(1 - \frac{\gamma+1}{2} \frac{g z}{c^2}\right) k_0 z\right)} + \mathcal{O}(\Gamma c^{-2}) \quad \text{with} \quad \mathcal{A} = \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix}$$

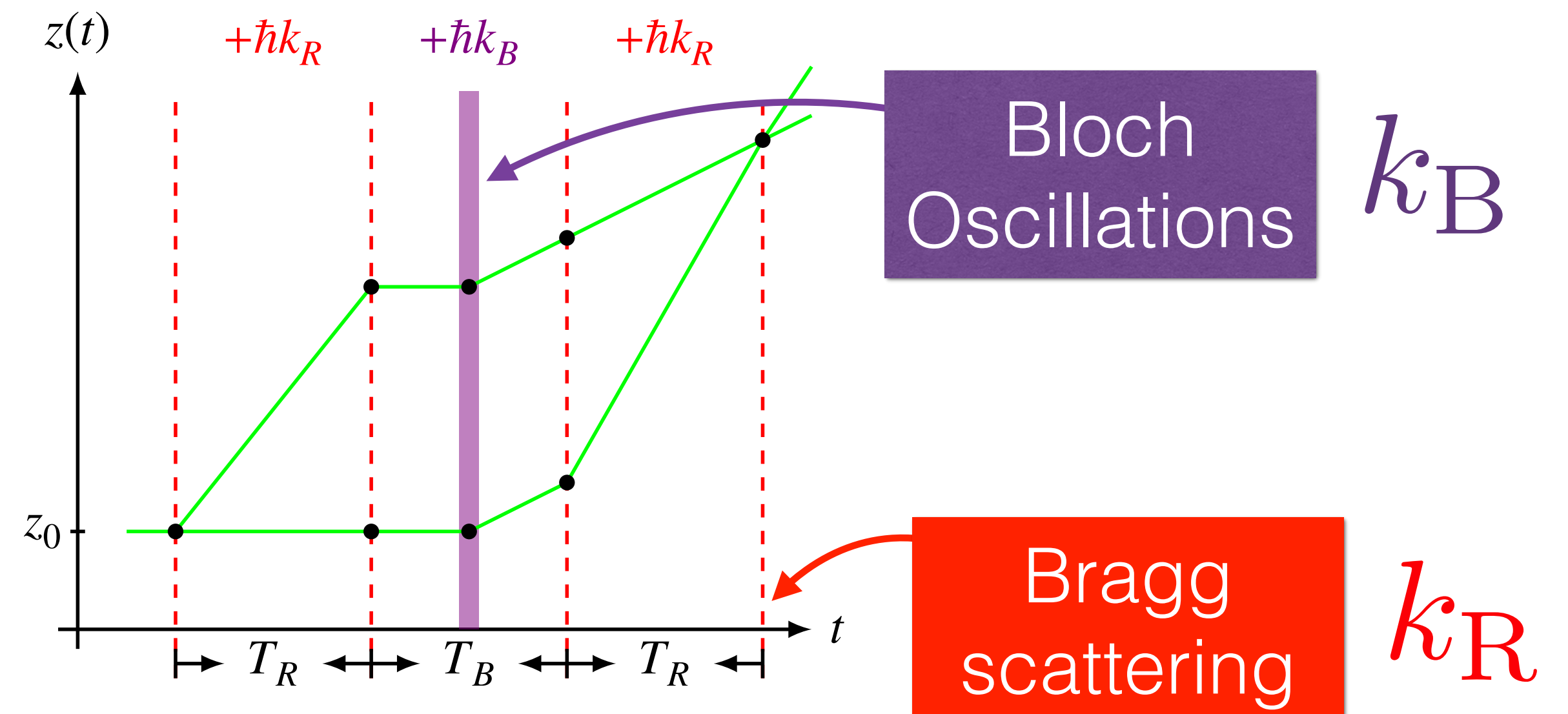
2. Systematic analysis of AIF classes



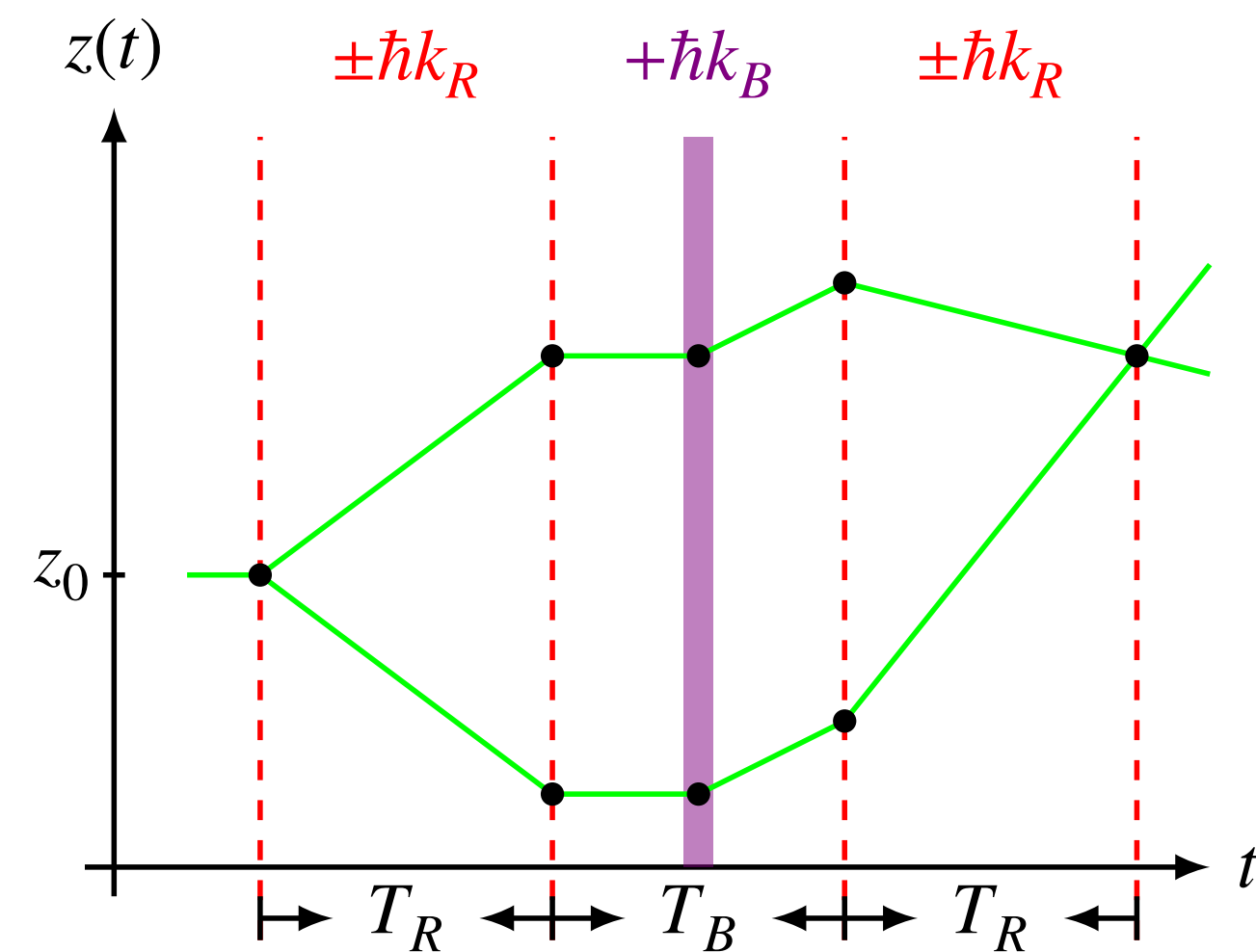
Interferometer geometries of interest



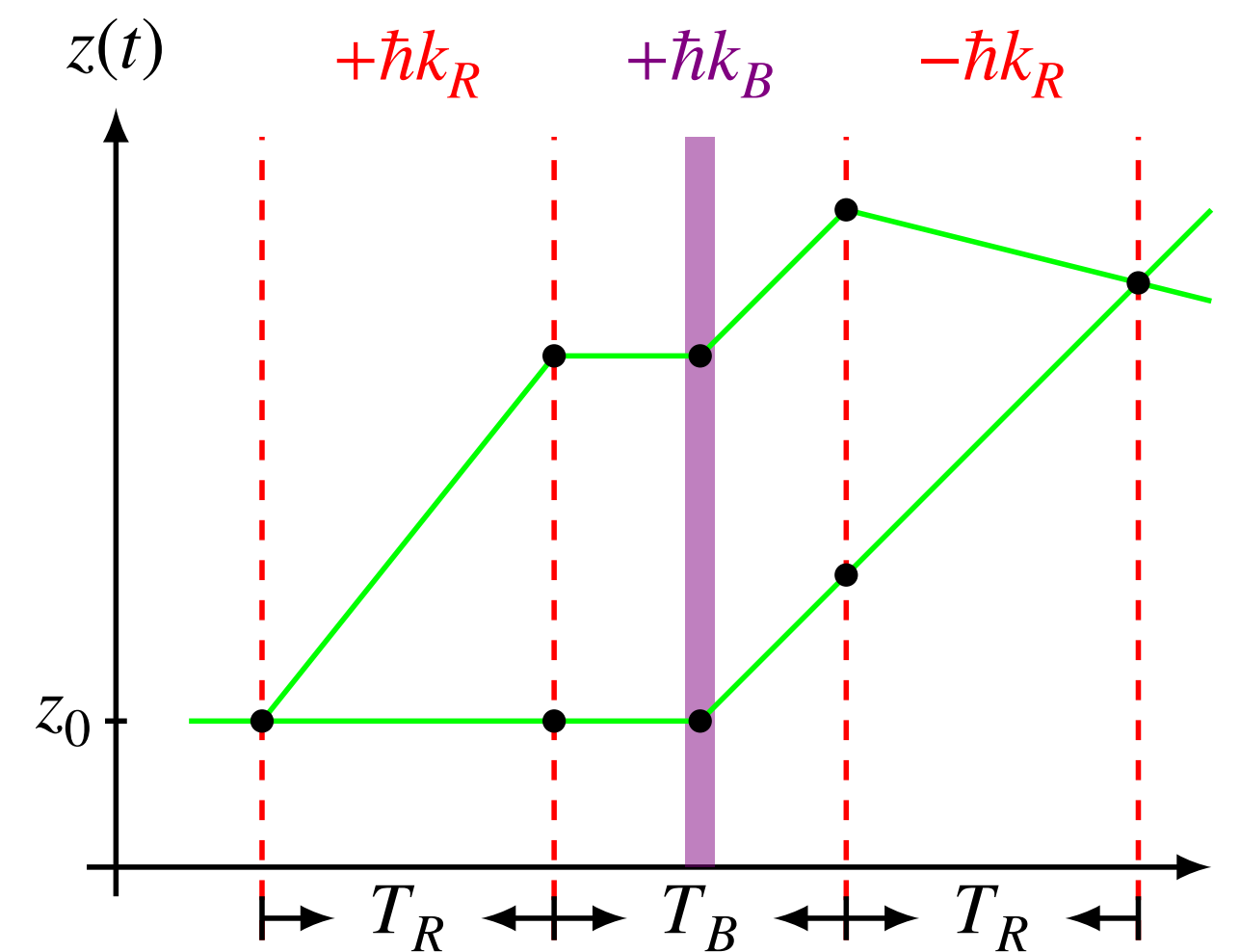
(a) Mach-Zehnder Interferometer (MZI)



(b) Symmetric Ramsey-Bordé (SRBI)



(c) Symmetric Double Diffraction (SDDI)



(d) Asymmetric Ramsey-Bordé (ARBI)

So many small parameters...

$$c^{-1}$$

$$\Gamma/c^2$$

$$\omega_R/c$$

$$\omega_R\Gamma/c$$

What shall we keep? Where should we truncate?

$$\Gamma^2/c$$

$$\Gamma/c$$

$$c^{-2}$$

$$\omega_R/c^2$$

Dimensionless description

$$\Delta\Phi = gk_R T_R^2 = \underbrace{\frac{mc^2}{\hbar} T_R}_{\sim 10^{-8}} \cdot \underbrace{\frac{gT_R}{c}}_{\sim 10^{-11}} \cdot \underbrace{\frac{\hbar k_R}{mc}}_{\sim 10^{-11}}$$

Denote this as $\mathcal{O}(2) \approx 10^7 \text{ rad}$

One can write **every** phase shift in an atom interferometer in this form!

Using this notation we can **systematically group** phase shifts!

$$\text{Different example: } \Delta\Phi = gk_R \Gamma_0 T_R^4 = \underbrace{\omega_C T_R}_{\sim 10^0} \cdot \underbrace{\frac{gT_R}{c}}_{\sim 10^{-11}} \cdot \underbrace{\frac{\hbar k_R}{mc}}_{\sim 10^{-11}} \cdot \underbrace{\Gamma T_R^2}_{\sim 10^0}$$

Denote this as $\mathcal{O}(3) \approx 10^0 \text{ rad}$

Dimensionless parameters (for now)

Parameter	Definition	
$\mathcal{Z}_0$	$\frac{z_0}{cT_R}$	Initial conditions: 10^{-8}
$\mathcal{V}_0$	$\frac{v_0}{c}$	
$\mathcal{F}_R$	$\frac{\hbar\omega_R}{mc^2}$	Recoil frequency: 10^{-20}
$\mathcal{G}_{1,i}$	$\frac{gT_i}{c}$	Gravitational potential: 10^{-8}
$\mathcal{G}_{2,i}$	ΓT_i^2	
$\mathcal{R}_i$	$\frac{\hbar k_i}{mc}$	Recoil momentum: $10^{-11} - 10^{-9}$

Smaller than the others

Python algorithm

Created an open source Python algorithm that symbolically (!) calculates the phase shifts using **SymPy**.

```
# ~~~~~  
# ~~ Mach-Zehnder Interferometer (MZI) (3-pulse Interferometer) ~~  
# ~~~~~
```

```
interfer  
time_dec
```

Let me show it to you!

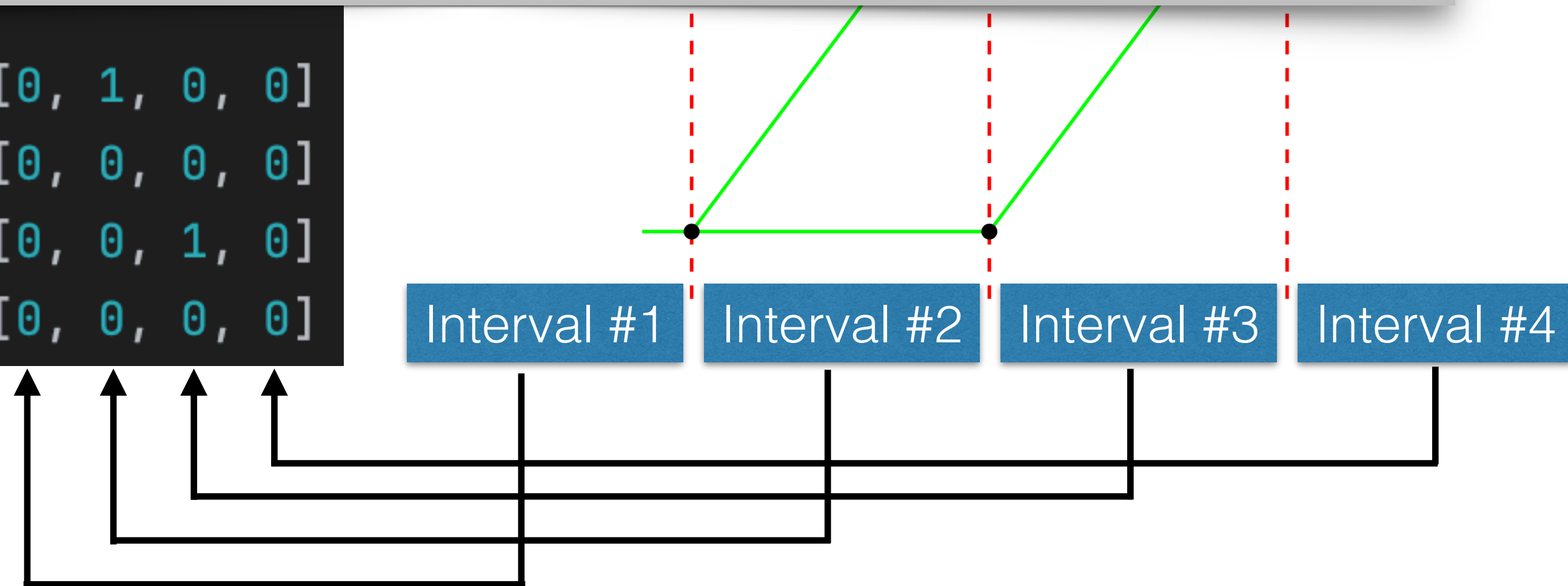
```
bragg_quanta_upper_path = [0, 1, 0, 0]  
bloch_quanta_upper_path = [0, 0, 0, 0]  
bragg_quanta_lower_path = [0, 0, 1, 0]  
bloch_quanta_lower_path = [0, 0, 0, 0]
```

Interval #1

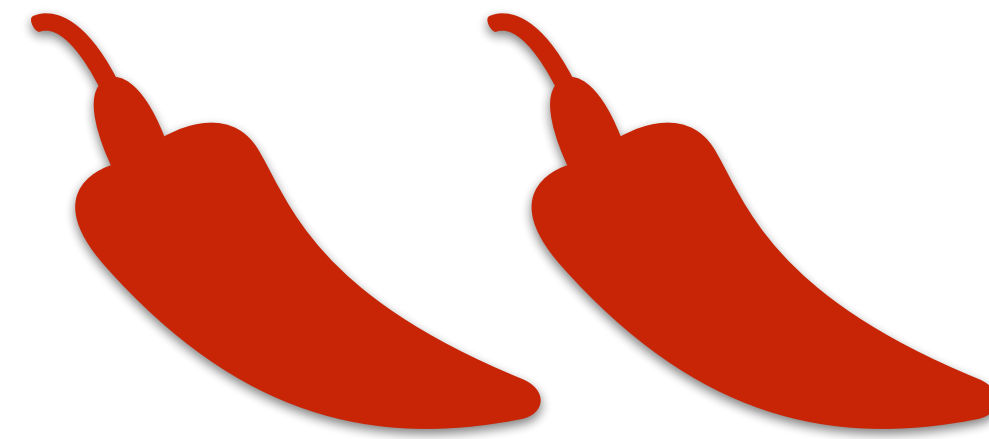
Interval #2

Interval #3

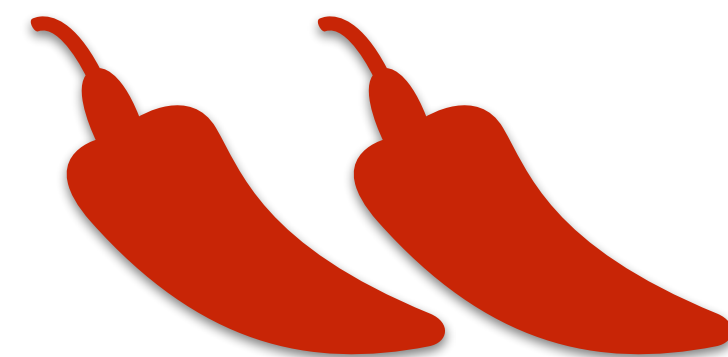
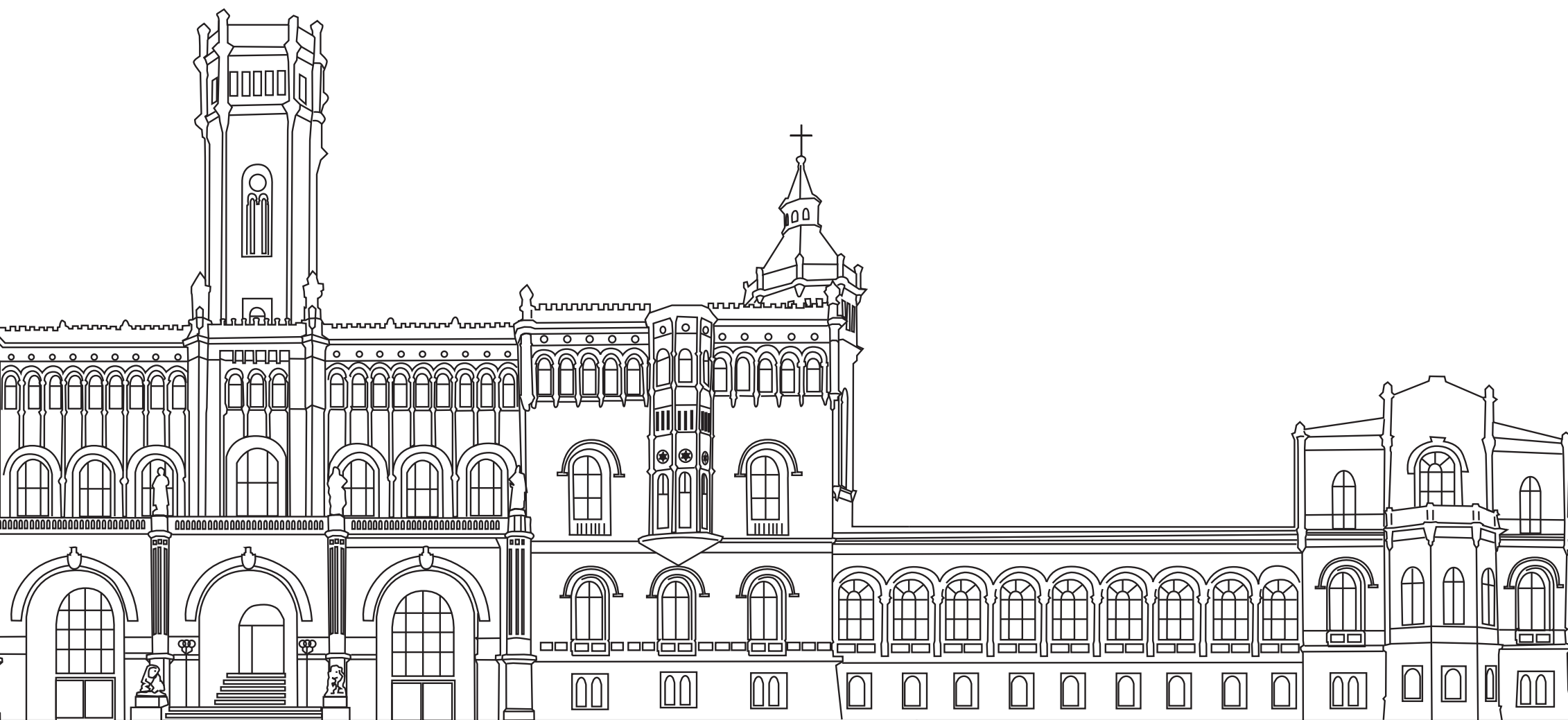
Interval #4



For the rest of today we'll
concentrate on spiciness level two:



3. Measurements of gravitational curvature



Phase shift results (without FSL)

Phases in units of ω_C							
#	Order	Proportionality	SRBI	SDDI	ARBI	α	Origin
1	$O(2)$	$\mathcal{G}_{1,R}\mathcal{R}_R$	$T_B + T_R$	$2T_B + 2T_R$	$(2T_B + 2T_R) \cdot \frac{T_B + T_R}{T_R}$	1	Non-relativistic $\mathcal{G}_{1,R}\mathcal{R}_R = 2gk_R T_R (T_B + T_R)$ $\alpha = 2$
2		$\mathcal{R}_R^2$	0	0			
3		$\mathcal{R}_R\mathcal{R}_B$	$-T_R$	$-2T_R$	$-T_R$	1	
4	$O(3)$	$\mathcal{R}_R\mathcal{Z}_0\mathcal{G}_{2,R}$	$-T_B - T_R$	$-2T_B - 2T_R$	$-T_B - T_R$	2	Gravity gradient $O(2)$
5		$\mathcal{R}_R\mathcal{V}_0\mathcal{G}_{2,R}$	$-\frac{3}{2}T_B - T_R$	$-3T_B - 2T_R$	$-\frac{3}{2}T_B - T_R$	3	
6		$\mathcal{R}_R\mathcal{R}_B\mathcal{G}_{2,R}$	$-\frac{1}{4}T_B - \frac{1}{6}T_R$	$-\frac{1}{2}T_B - \frac{1}{3}T_R$	$-\frac{1}{4}T_B - \frac{1}{6}T_R$	4	
7		$\mathcal{R}_R\mathcal{G}_{1,R}\mathcal{G}_{2,R}$	$\frac{7}{6}T_B + \frac{7}{12}T_R$	$\frac{7}{3}T_B + \frac{7}{6}T_R$	$\frac{7}{6}T_B + \frac{7}{12}T_R$	4	
8		$\mathcal{R}_R^2\mathcal{G}_{2,R}$	$-\frac{1}{2}T_B - \frac{1}{2}T_R$	0	$-\frac{1}{2}T_B - \frac{1}{3}T_R$	3	
9		$\mathcal{R}_R\mathcal{V}_0\mathcal{G}_{2,B}$	$-\frac{1}{2}T_R$	$-T_R$	$-\frac{1}{2}T_R$	3	
10		$\mathcal{R}_R\mathcal{R}_B\mathcal{G}_{2,B}$	$-\frac{1}{8}T_R$	$-\frac{1}{4}T_R$	$-\frac{1}{8}T_R$	3	
11		$\mathcal{R}_R\mathcal{G}_{1,R}\mathcal{G}_{2,B}$	$\frac{1}{6}T_B + \frac{3}{4}T_R$	$\frac{1}{3}T_B + \frac{3}{2}T_R$	$\frac{1}{6}T_B + \frac{3}{4}T_R$	4	
12	Small	$\mathcal{F}_R\mathcal{G}_{1,R}^2$	$-\frac{9}{2}T_B - 3T_R$	$-9T_B - 6T_R$	$-\frac{9}{2}T_B - 3T_R$	3	Doppler effect
13		$\mathcal{F}_R\mathcal{G}_{1,R}\mathcal{V}_0$	$3T_B + 3T_R$	$6T_B + 6T_R$	$3T_B + 3T_R$	2	
14		$\mathcal{F}_R\mathcal{G}_{1,R}\mathcal{G}_{1,B}$	$-\frac{3}{2}T_B$	$-3T_B$	$-\frac{3}{2}T_B$	3	
15		$\mathcal{F}_R\mathcal{R}_B\mathcal{G}_{1,R}$	$\frac{5}{2}T_B + \frac{7}{2}T_R$	$5T_B + 7T_R$	$\frac{5}{2}T_B + \frac{7}{2}T_R$	2	
16		$\mathcal{F}_R\mathcal{R}_B^2$	$-T_R$	$-2T_R$	$-T_R$	1	
17		$\mathcal{F}_R\mathcal{R}_B\mathcal{V}_0$	$-2T_R$	$-4T_R$	$-2T_R$	1	
18		$\mathcal{F}_R\mathcal{R}_R^2$	0	$2T_R$	T_R	1	
19		$\mathcal{F}_R\mathcal{R}_R\mathcal{G}_{1,B}$	$\frac{1}{2}T_B$	0	$-\frac{1}{2}T_B$	2	
20		$\mathcal{F}_R\mathcal{R}_R\mathcal{G}_{1,R}$	$3T_B - \frac{5}{2}T_R$	0	$-3T_B + \frac{9}{2}T_R$	2	
21		$\mathcal{F}_R\mathcal{R}_R\mathcal{R}_B$	$-\frac{1}{2}T_B + 2T_R$	0	$\frac{1}{2}T_B - 2T_R$	1	
22		$\mathcal{F}_R\mathcal{R}_R\mathcal{V}_0$	$-T_B - T_R$	0	$T_B + 5T_R$	1	
23		$\mathcal{F}_R\mathcal{R}_R\mathcal{Z}_0$	0	0	$2T_R$	1	

Phase shift results (without FSL)

Proportionality	SRBI	SDDI	ARBI	α
$\mathcal{G}_{1,R}\mathcal{R}_R$	$T_B + T_R$	$2T_B + 2T_R$	$T_B + T_R$	2
$\mathcal{R}_R^2$	0	0	T_R	1
$\mathcal{R}_R\mathcal{R}_B$	$-T_R$	$-2T_R$	$-T_R$	1
$\mathcal{R}_R\mathcal{Z}_0\mathcal{G}_{2,R}$	$-T_B - T_R$	$-2T_B - 2T_R$	$-T_B - T_R$	2
$\mathcal{R}_R\mathcal{V}_0\mathcal{G}_{2,R}$	$-\frac{3}{2}T_B - T_R$	$-3T_B - 2T_R$	$-\frac{3}{2}T_B - T_R$	3
$\mathcal{R}_R\mathcal{R}_B\mathcal{G}_{2,R}$	$-\frac{1}{4}T_B - \frac{1}{6}T_R$	$-\frac{1}{2}T_B - \frac{1}{3}T_R$	$-\frac{1}{4}T_B - \frac{1}{6}T_R$	4
$\mathcal{R}_R\mathcal{G}_{1,R}\mathcal{G}_{2,R}$	$\frac{7}{6}T_B + \frac{7}{12}T_R$	$\frac{7}{3}T_B + \frac{7}{6}T_R$	$\frac{7}{6}T_B + \frac{7}{12}T_R$	4
$\mathcal{R}_R^2\mathcal{G}_{2,R}$	$-\frac{1}{2}T_B - \frac{1}{2}T_R$	0	$-\frac{1}{2}T_B - \frac{1}{3}T_R$	3
$\mathcal{R}_R\mathcal{V}_0\mathcal{G}_{2,B}$	$-\frac{1}{2}T_R$	$-T_R$	$-\frac{1}{2}T_R$	3
$\mathcal{R}_R\mathcal{R}_B\mathcal{G}_{2,B}$	$-\frac{1}{8}T_R$	$-\frac{1}{4}T_R$	$-\frac{1}{8}T_R$	3
$\mathcal{R}_R\mathcal{G}_{1,R}\mathcal{G}_{2,B}$	$\frac{1}{6}T_B + \frac{3}{4}T_R$	$\frac{1}{3}T_B + \frac{3}{2}T_R$	$\frac{1}{6}T_B + \frac{3}{4}T_R$	4

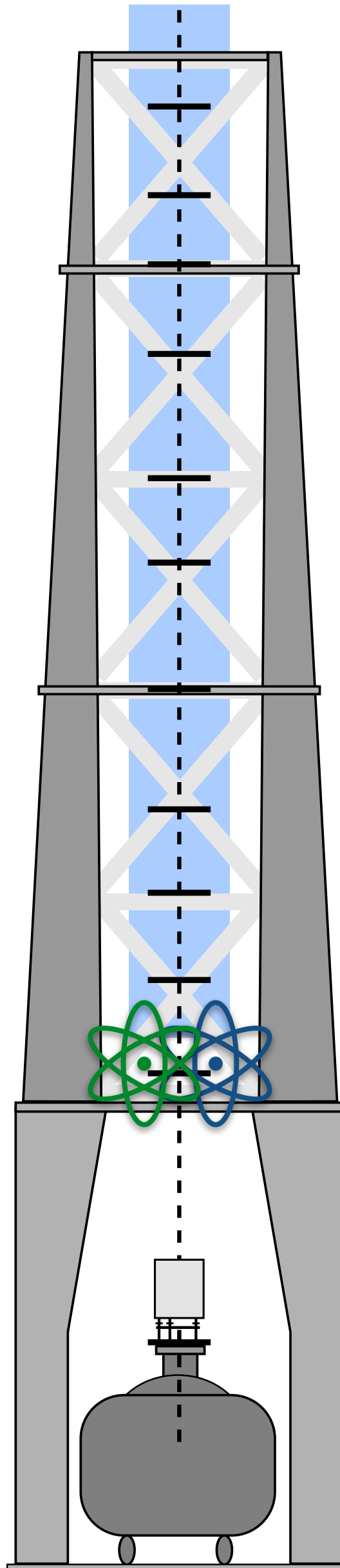
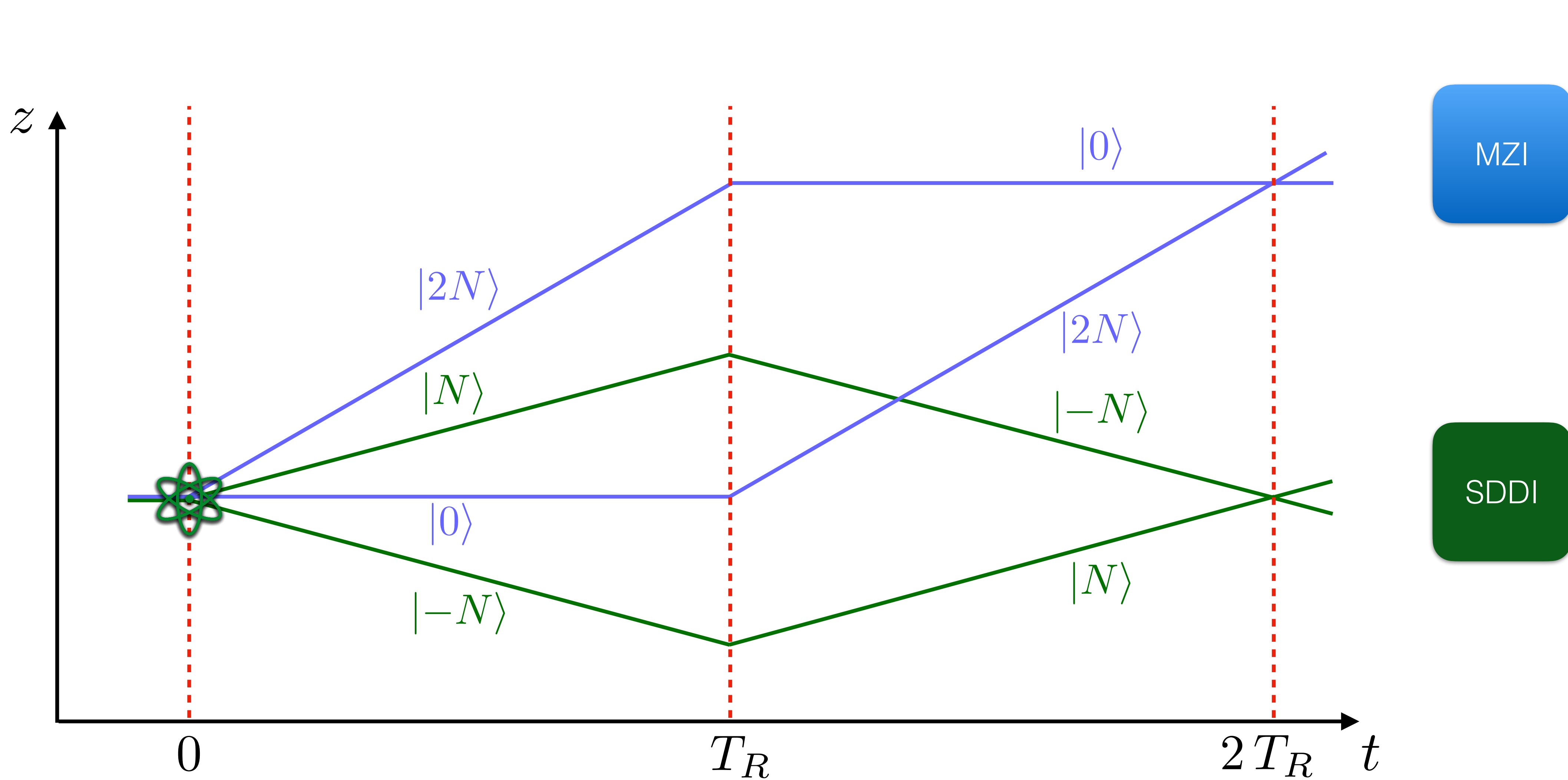
Interesting asymmetry

$$\omega_C T_R \cdot \mathcal{R}_R^2 \mathcal{G}_{2,R} = \frac{\hbar k_R^2 \Gamma_0 T_R^3}{m}$$

Most phases scale
with a factor of two
(spacetime area).
But this one not!

Can we use this to
gain information about
the gravity gradient?

Co-located Gradiometric Interferometer (CGI)



VLBAI figure adapted from Schilling et al. „Gravity field modelling for the Hannover 10 m atom interferometer“

CGI phase comparison

Differential phase shift is proportional to the gravity gradient

$$\Delta\Phi = -2N^2\mathcal{G}_{2,R}\mathcal{R}_R^2 = -2\frac{N^2\hbar k_R^2\Gamma_0 T_R^3}{m} = f \cdot \Gamma_0$$

Phase comparison of a MZI and a SDDI in a CGI configuration (no FSL phases)					
MZI	SDDI	Phase	Phase	Magnitude [rad]	Differential signal
2	2	$Nk_R g T_R^2$	$N\mathcal{R}_R\mathcal{G}_{1,R}$	1.4×10^7	0
-2	-2	$Nk_R z_0 \Gamma_0 T_R^2$	$N\mathcal{Z}_0\mathcal{G}_{2,R}\mathcal{R}_R$	20	0
-2	-2	$Nk_R v_0 \Gamma_0 T_R^3$	$N\mathcal{V}_0\mathcal{G}_{2,R}\mathcal{R}_R$	14	0
$\frac{7}{6}$	$\frac{7}{6}$	$Nk_R g \Gamma_0 T_R^4$	$N\mathcal{G}_{1,R}\mathcal{G}_{2,R}\mathcal{R}_R$	14	0
-2	0	$\frac{N^2\hbar k_R^2\Gamma_0 T_R^3}{m}$	$N^2\mathcal{G}_{2,R}\mathcal{R}_R^2$	1.5×10^{-2}	-2
-12	-12	$\frac{N\omega_R g^2 T_R^3}{c^2}$	$N\mathcal{G}_{1,R}^2\mathcal{F}_R$	2.3×10^{-9}	0
12	12	$\frac{N\omega_R g v_0 T_R^2}{c^2}$	$N\mathcal{F}_R\mathcal{G}_{1,R}\mathcal{V}_0$	2.4×10^{-9}	0

$$f = -2\frac{N^2\hbar k_R^2 T_R^3}{m}$$

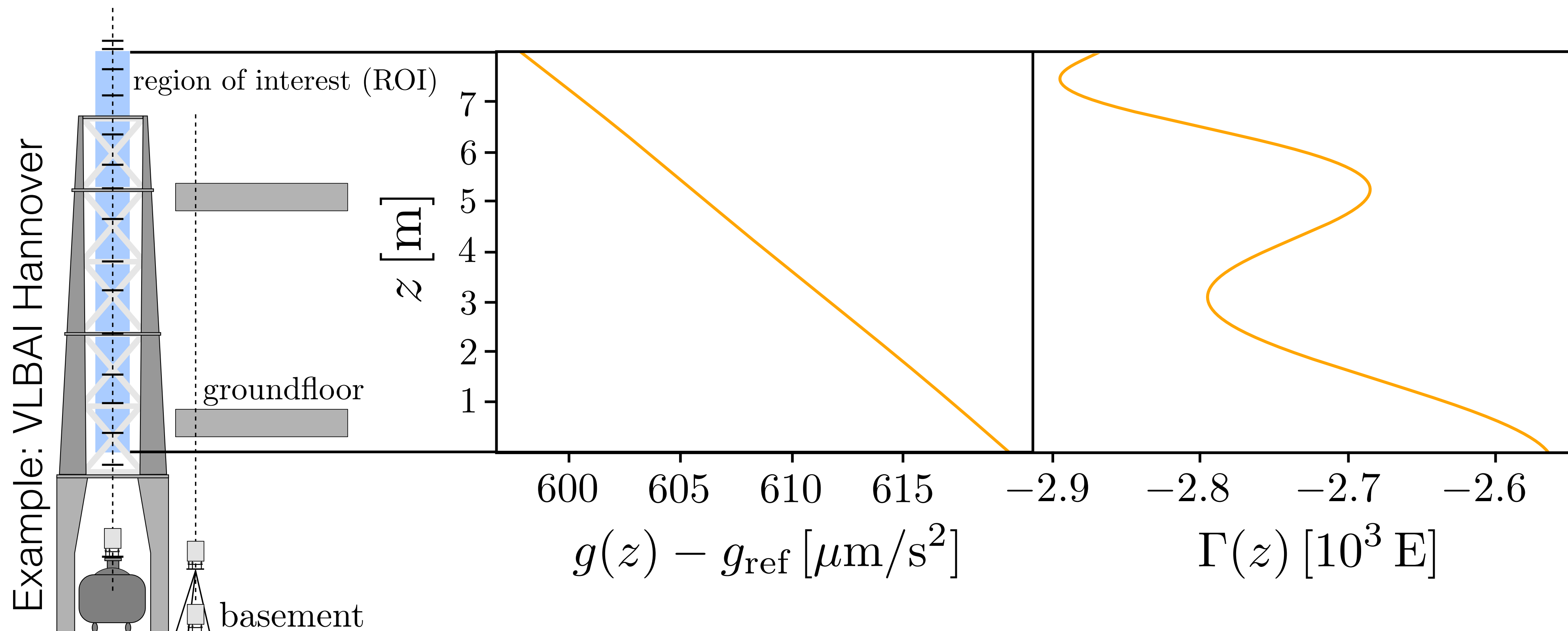
Looks interesting so far, but the
gravitational field is highly
idealised...

Complex gravitational fields

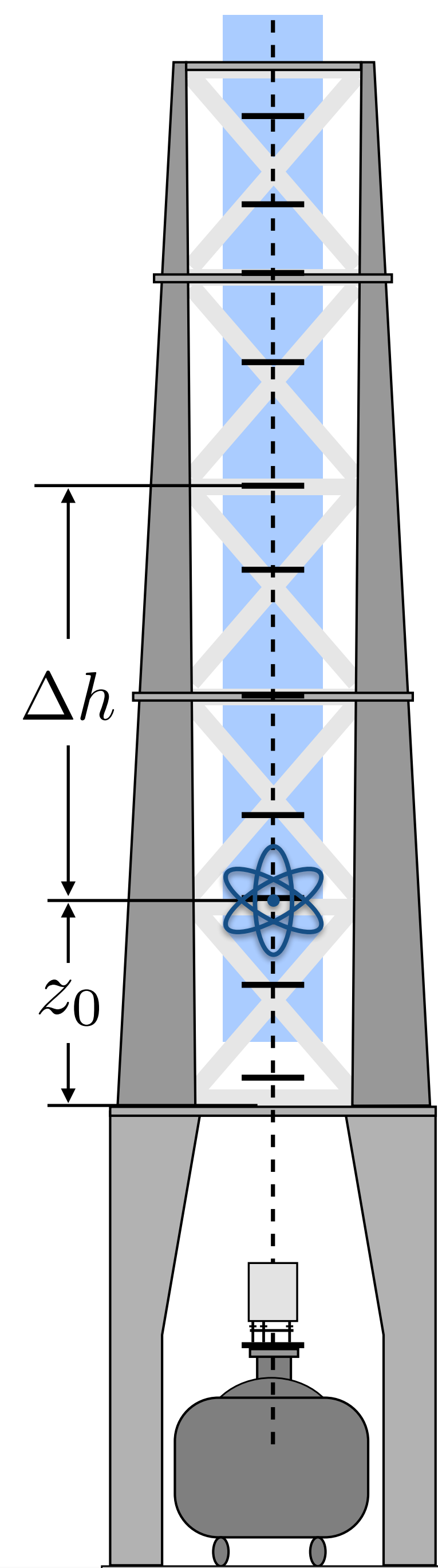
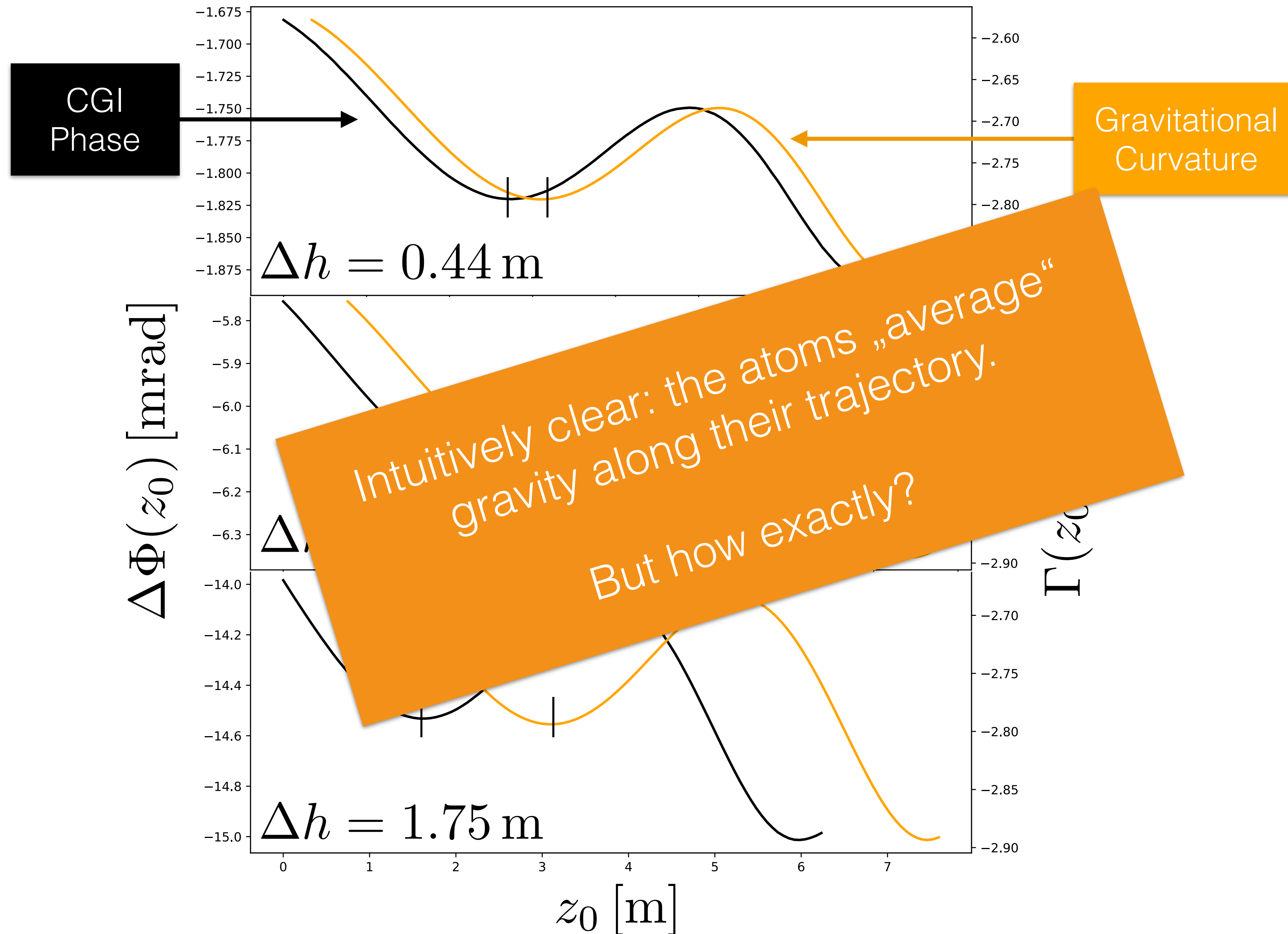
Consider $\phi(z) = \phi_0 + gz + \sum_{n=2} \frac{\phi^{(n)}}{n!} z^n \dots$

$$\begin{aligned} \phi^{(2)} &= -\Gamma_0 \\ \phi^{(3)} &= \Lambda_0 \end{aligned}$$

with a non-trivial $\Gamma(z) = \partial_z^2 \phi(z) = \text{Gravitational curvature}$



CGI: Numerical simulation in VLBAI



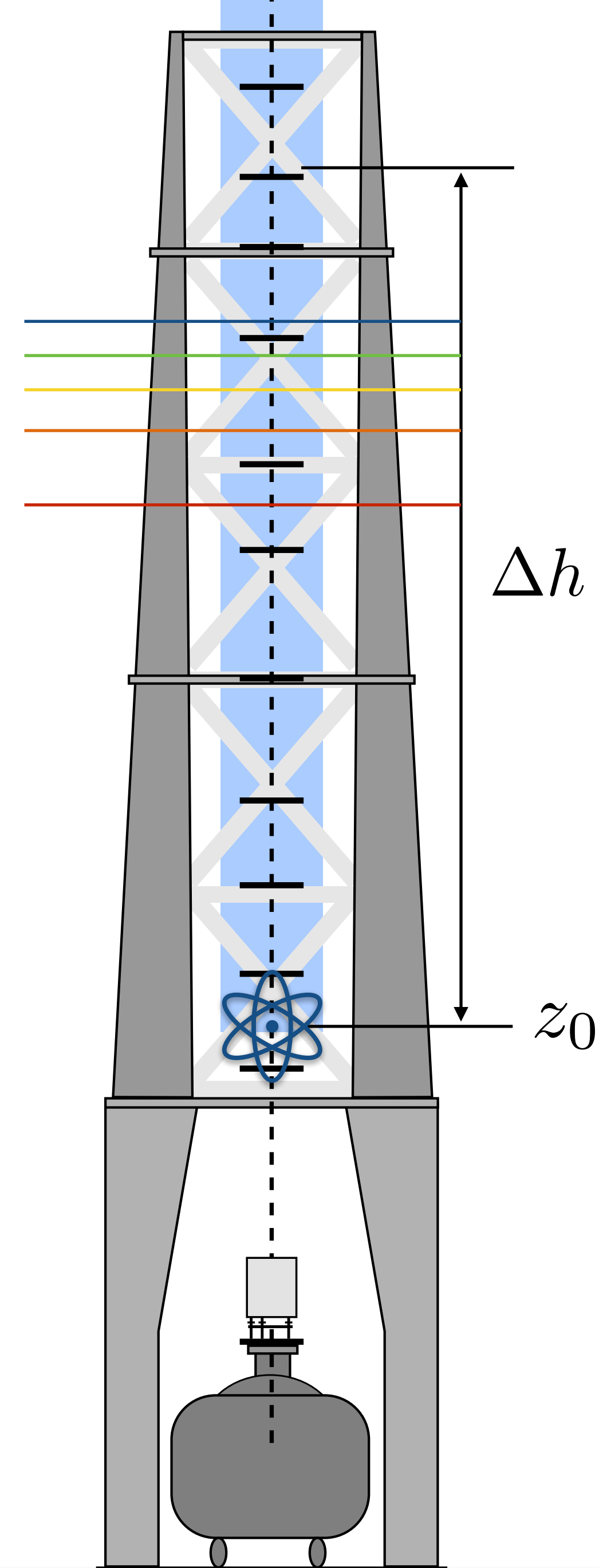
Averaging gravity

Where are the atoms „on average“?

Answer: It depends on the averaging process!

$$\|z(t)\|_n = \left(\frac{1}{2T_R} \int_0^{2T_R} |z(t) - z_0|^n dt \right)^{1/n}$$

n	1	2	3	4	5
$\ z(t)\ _n$	$0.66 \Delta h$	$0.73 \Delta h$	$0.77 \Delta h$	$0.79 \Delta h$	$0.82 \Delta h$



Estimator for grav. curvature

We can now define a novel estimator $\hat{\Gamma}(z_0)$ for the grav. curvature:

Step 1: Convert phase to grav. curvature using the scale factor.

$$\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0)}{f}$$

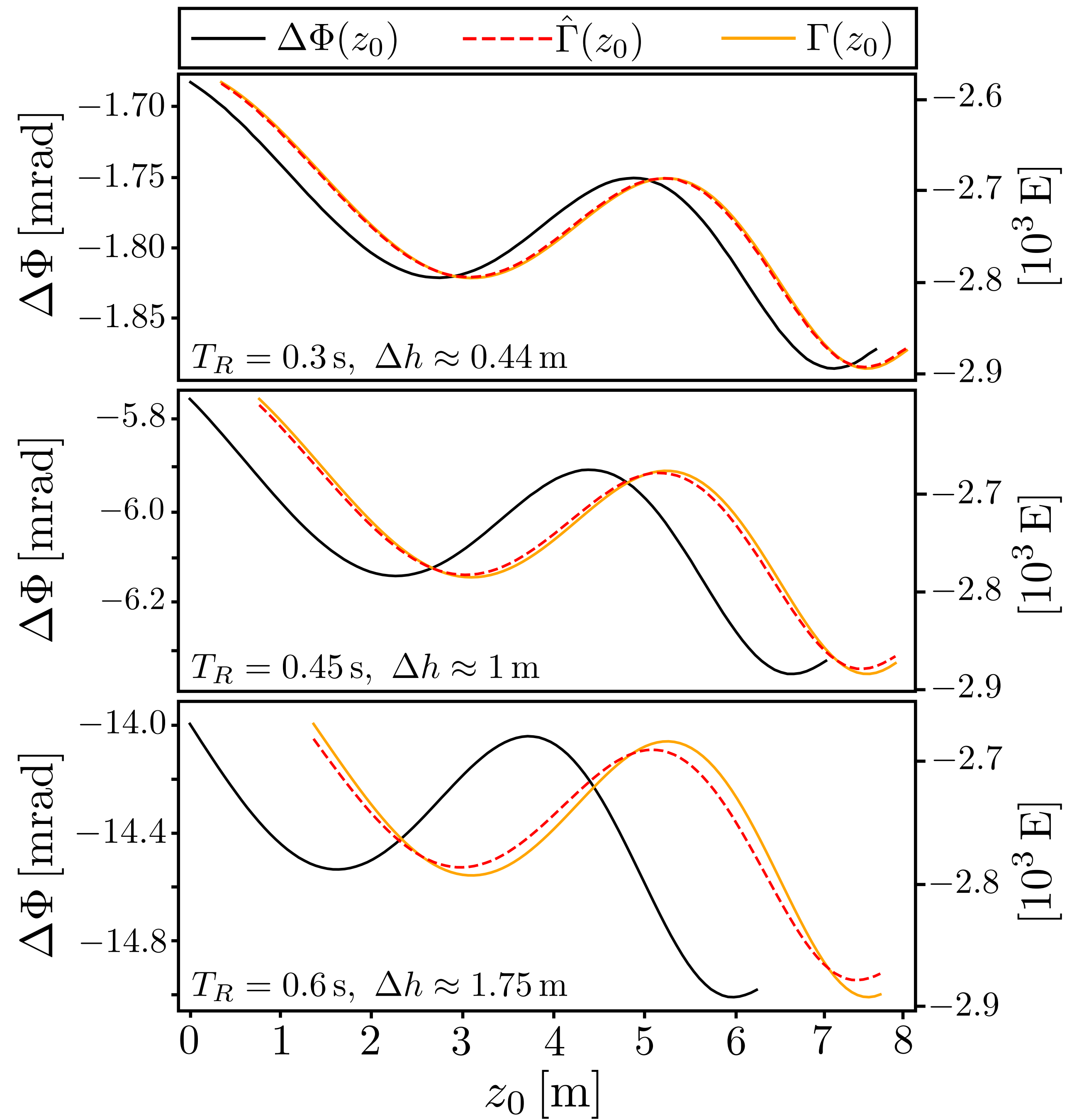
Step 2: Shift the height via the **cubic** mean of the trajectory.

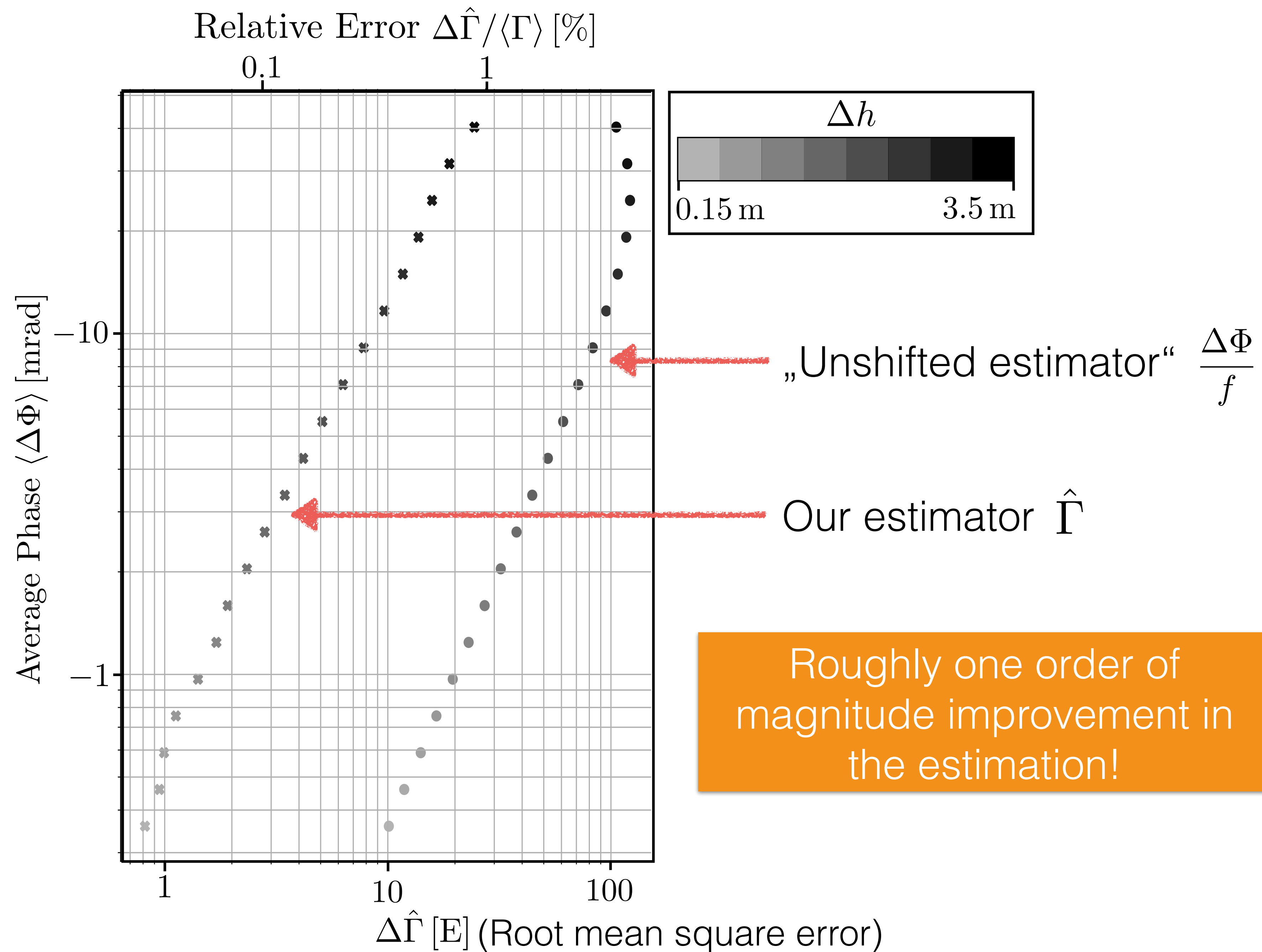
$$\hat{\Gamma}(z_0) = \frac{\Delta\Phi(z_0 - \|z(t)\|_3)}{f}$$

Reminder:

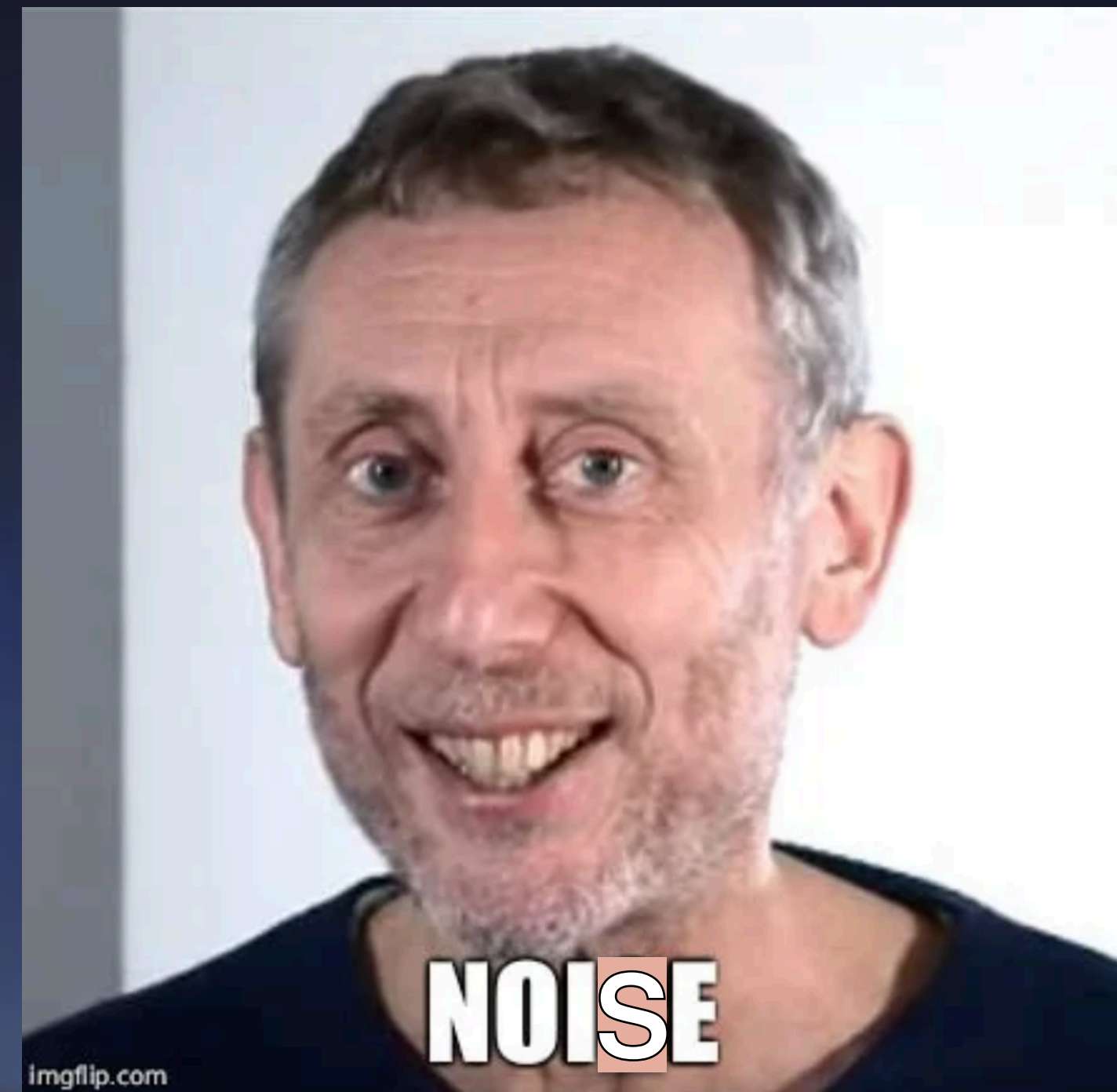
$$f = -2 \frac{N^2 \hbar k_R^2 T_R^3}{m}$$

Motivation for the **cubic** mean.

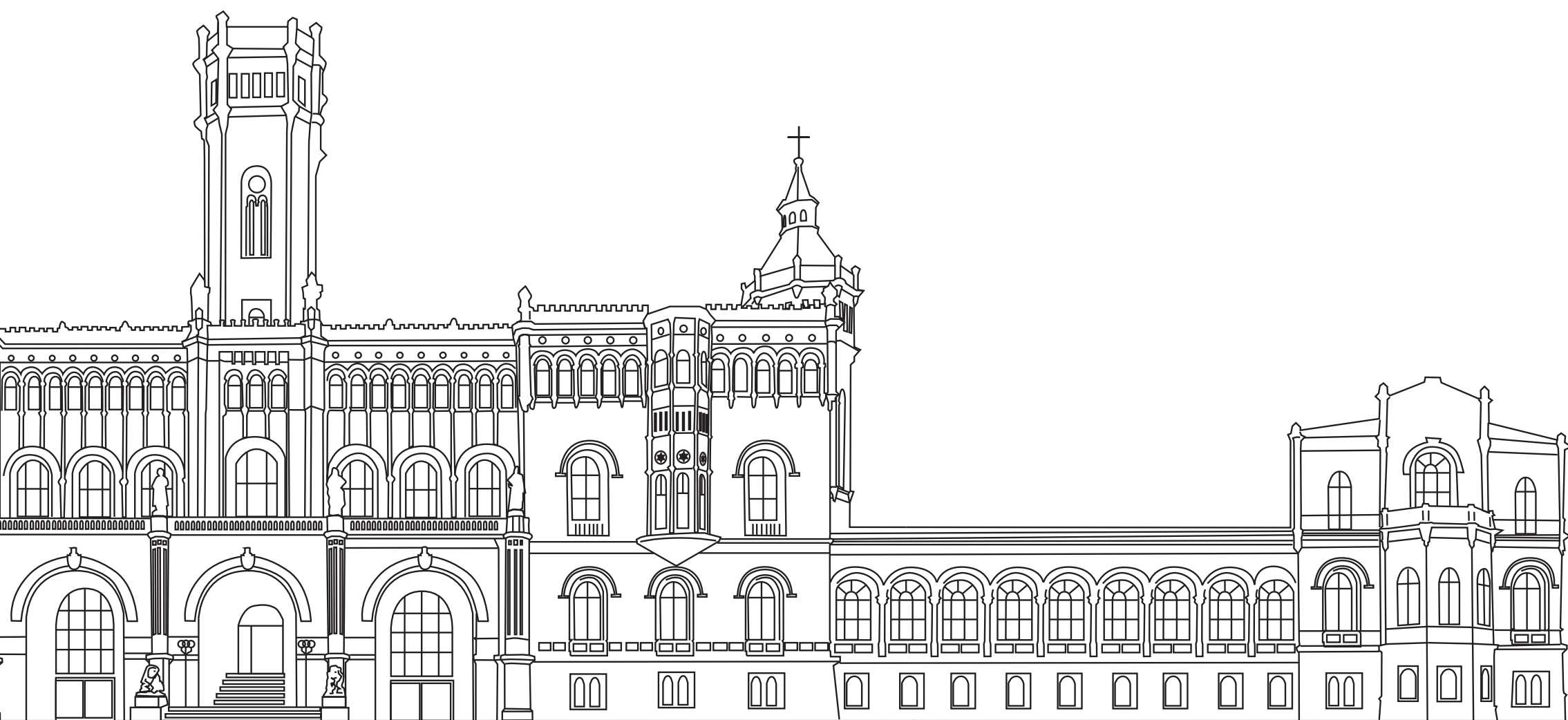




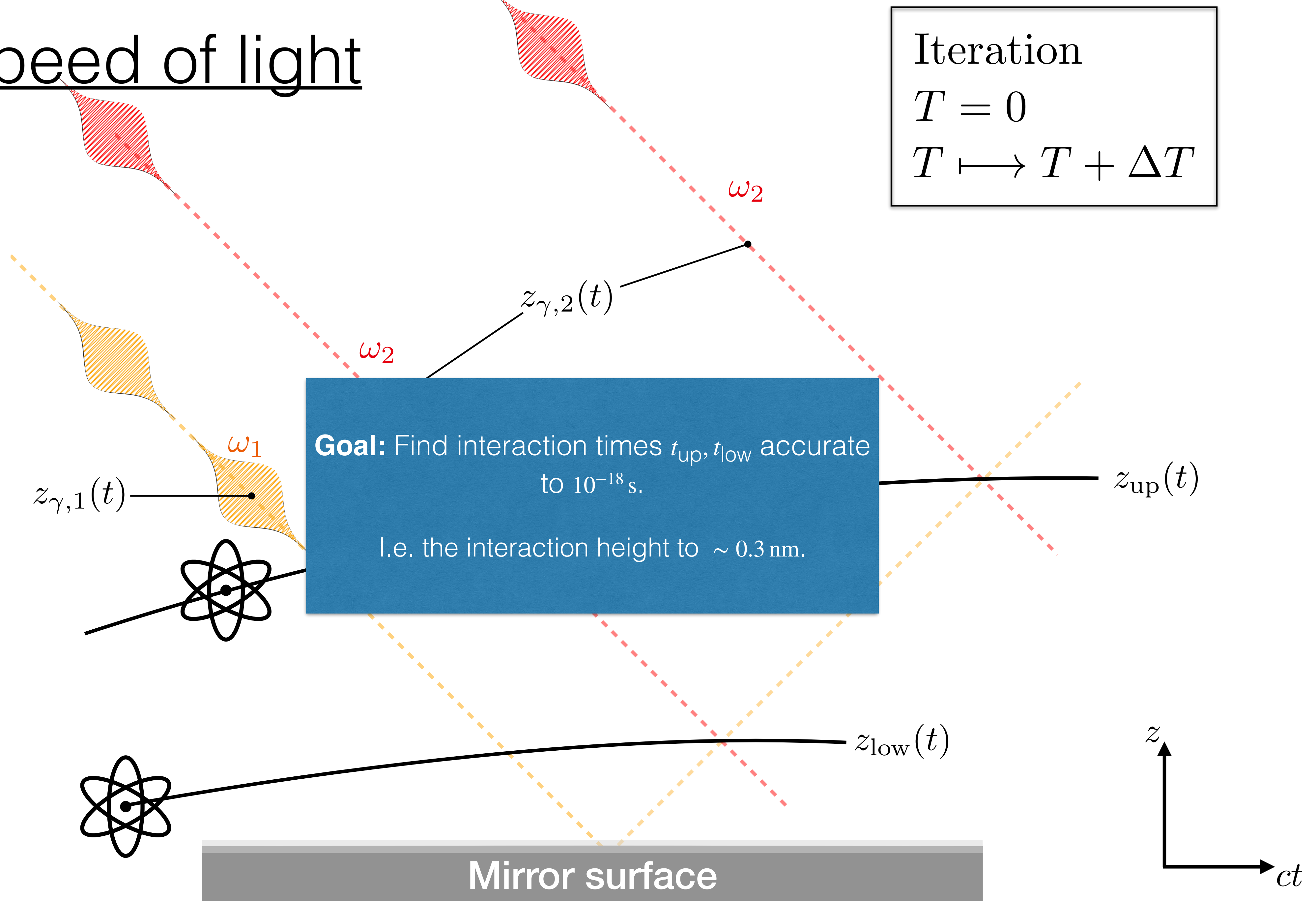
One more thing...



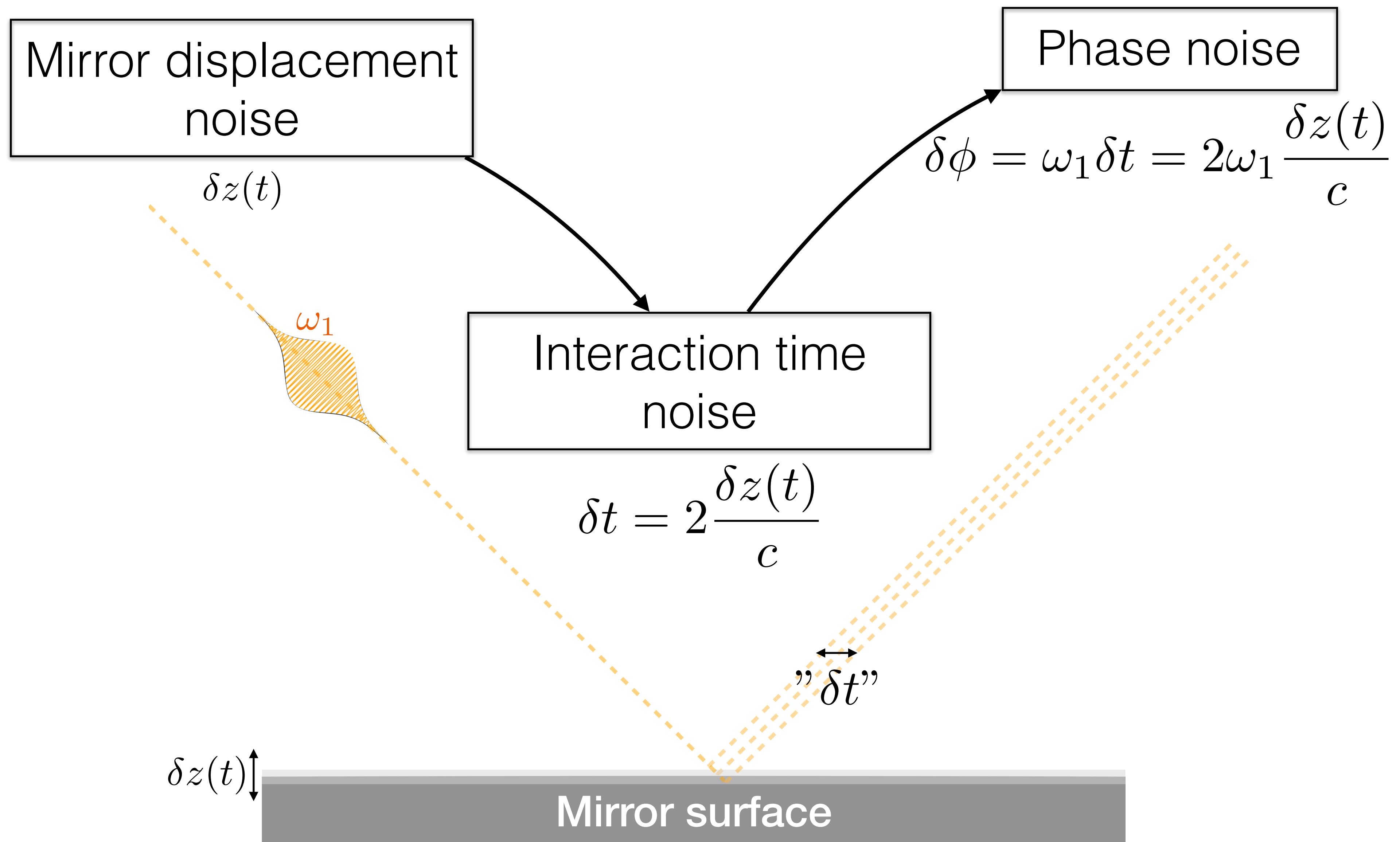
4. Noise



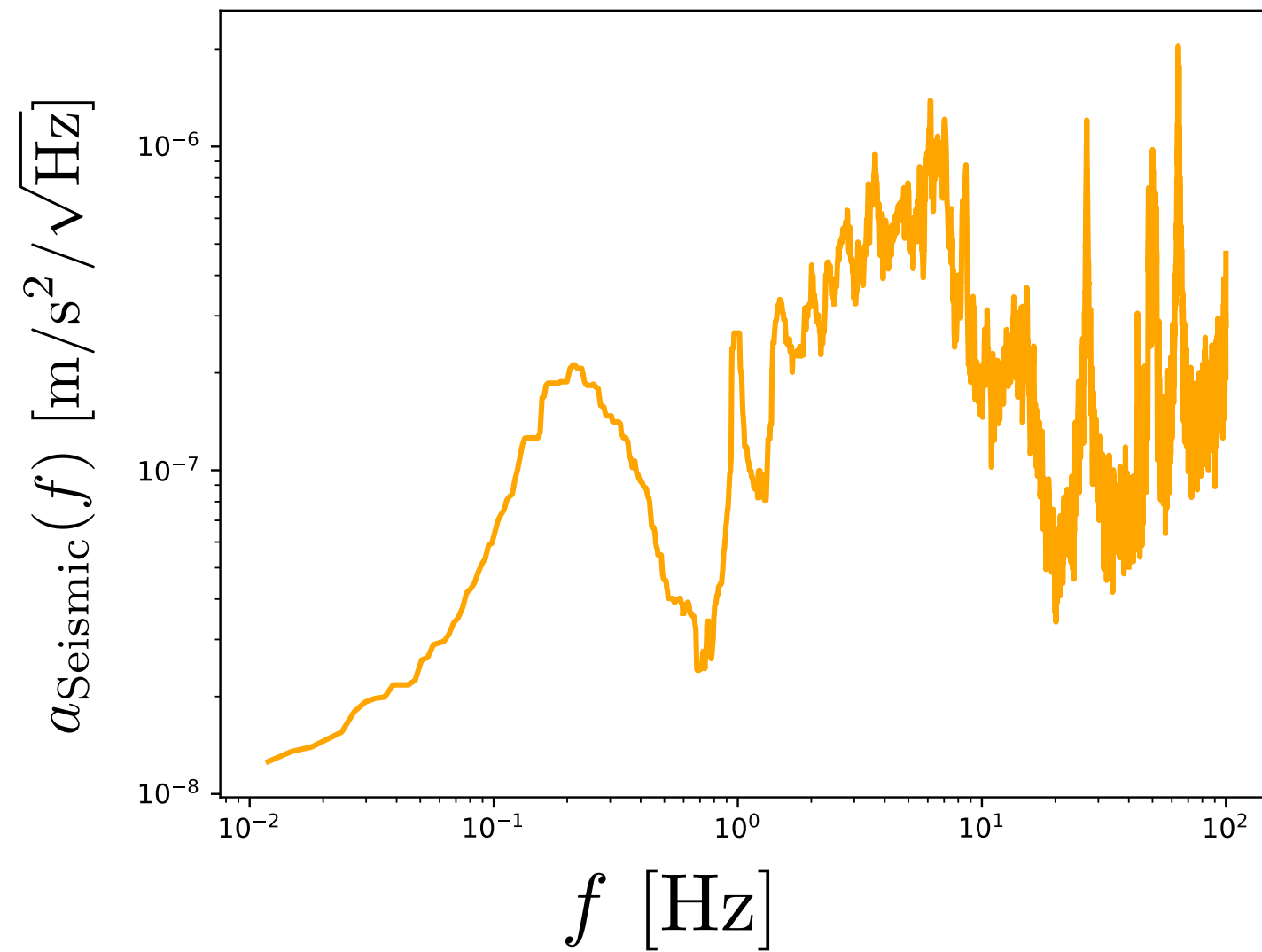
Finite speed of light



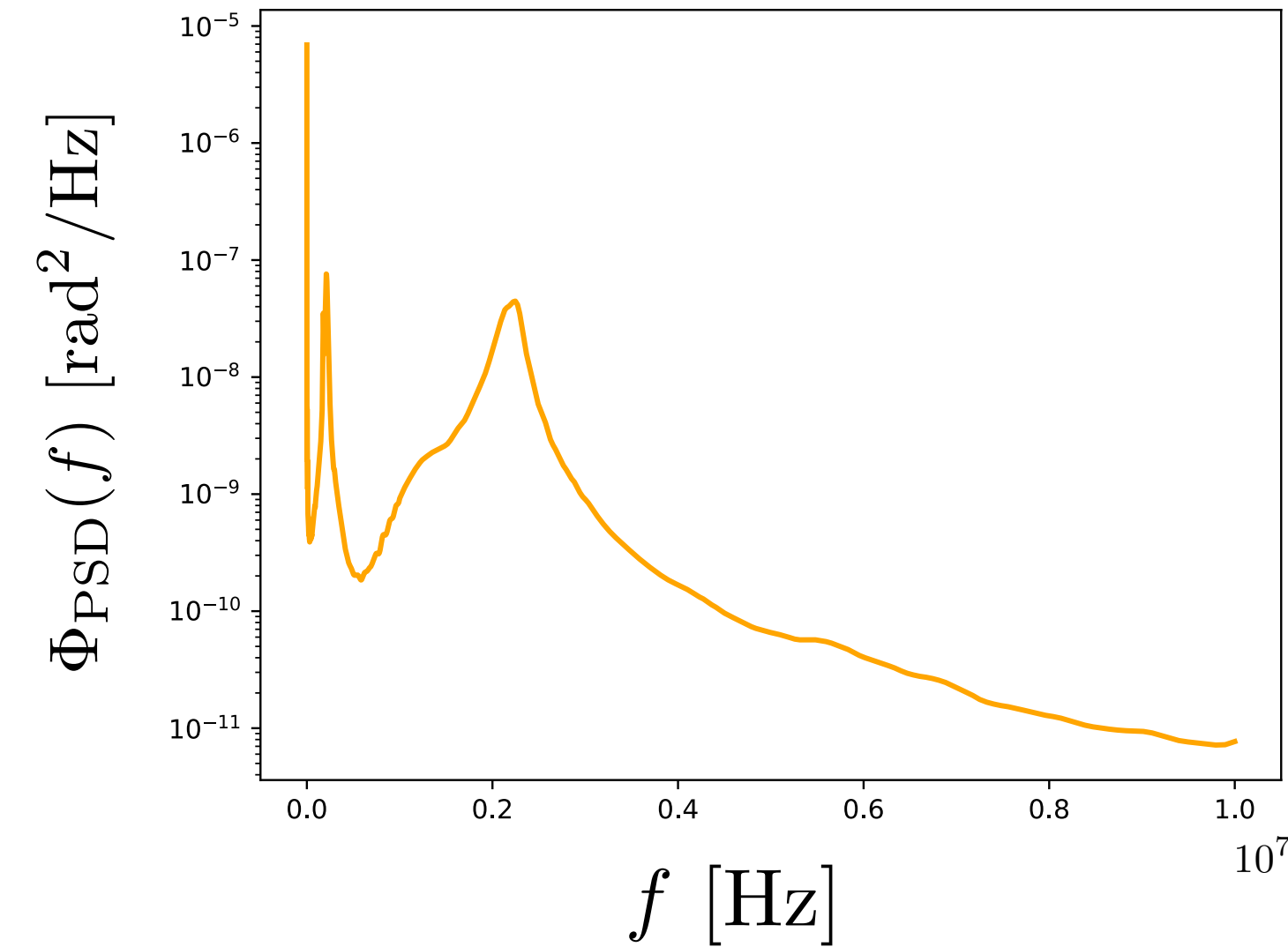
Finite speed of light



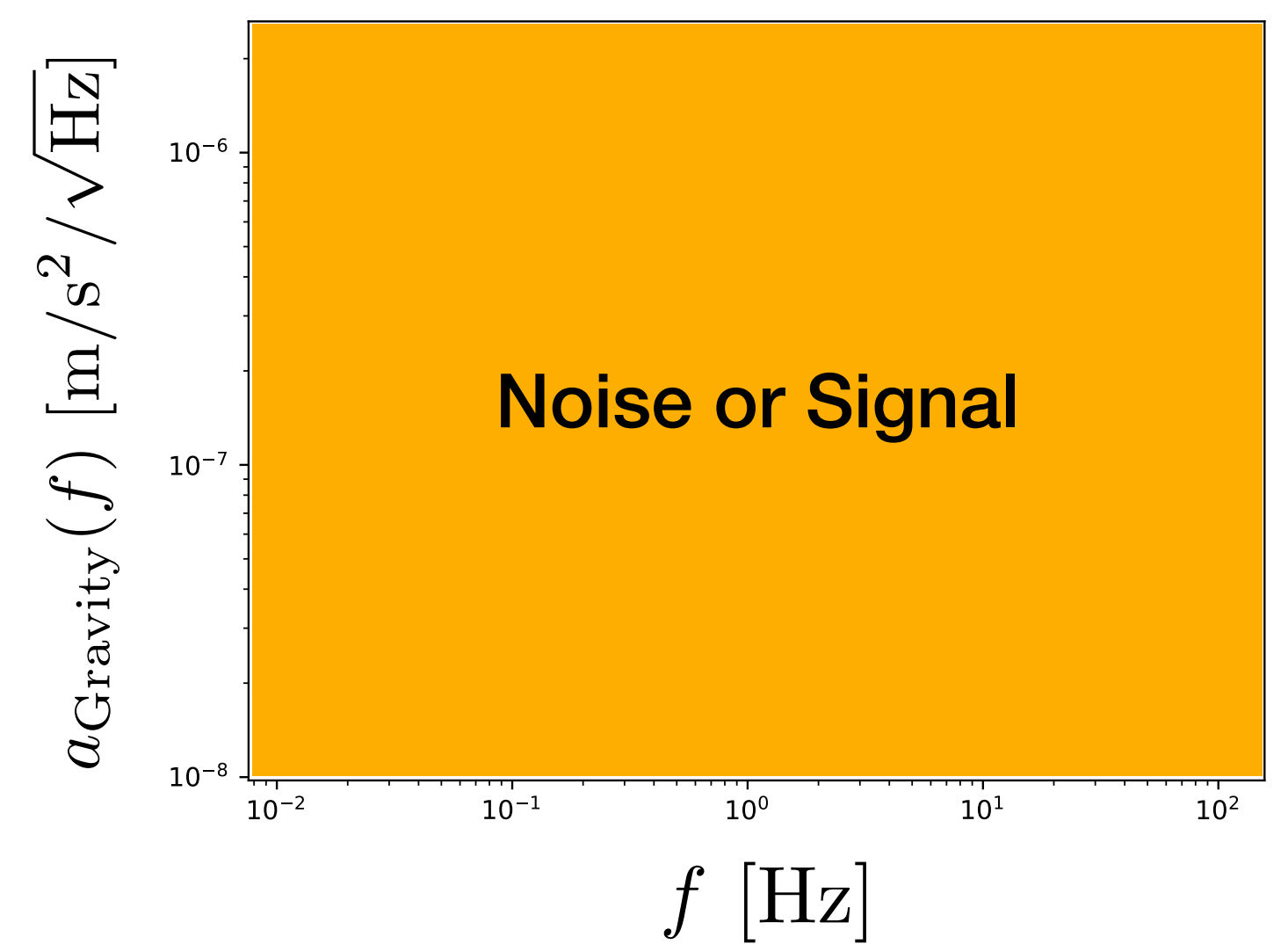
1. Seismic Acceleration



2. Laser Phase Noise



3. Gravity Noise



4. Discretize

Choose frequencies linearly in the respective intervals

$$f_i \in [10^{-2} \text{ Hz}, 10^2 \text{ Hz}]$$

$$f_j \in [1 \text{ Hz}, 10^7 \text{ Hz}]$$

$$f_k \in [10^{-2} \text{ Hz}, 10^2 \text{ Hz}]$$

for i, j, k

5. Create noise

Repeat N_S times

Sample uniformly $\Phi_i, \Phi_j, \Phi_k \in [0, 2\pi)$

$$\delta z(t) = - \sum_i \frac{a_{\text{Seismic}}(f_i)}{\sqrt{(2\pi f_i)^3}} \cdot \sin(2\pi f_i t + \Phi_i)$$

$$\delta \Phi(t) = \sum_j \sqrt{2\pi f_j \Phi_{\text{PSD}}(f_j)} \cdot \sin(2\pi f_j t + \Phi_j)$$

$$\delta g(t) = \sum_k a_{\text{Gravity}}(f_k) \cdot \sqrt{2\pi f_k} \cdot \sin(2\pi f_k t + \Phi_k)$$

$$\delta \phi(t) = -R_{\oplus} \delta g(t)$$

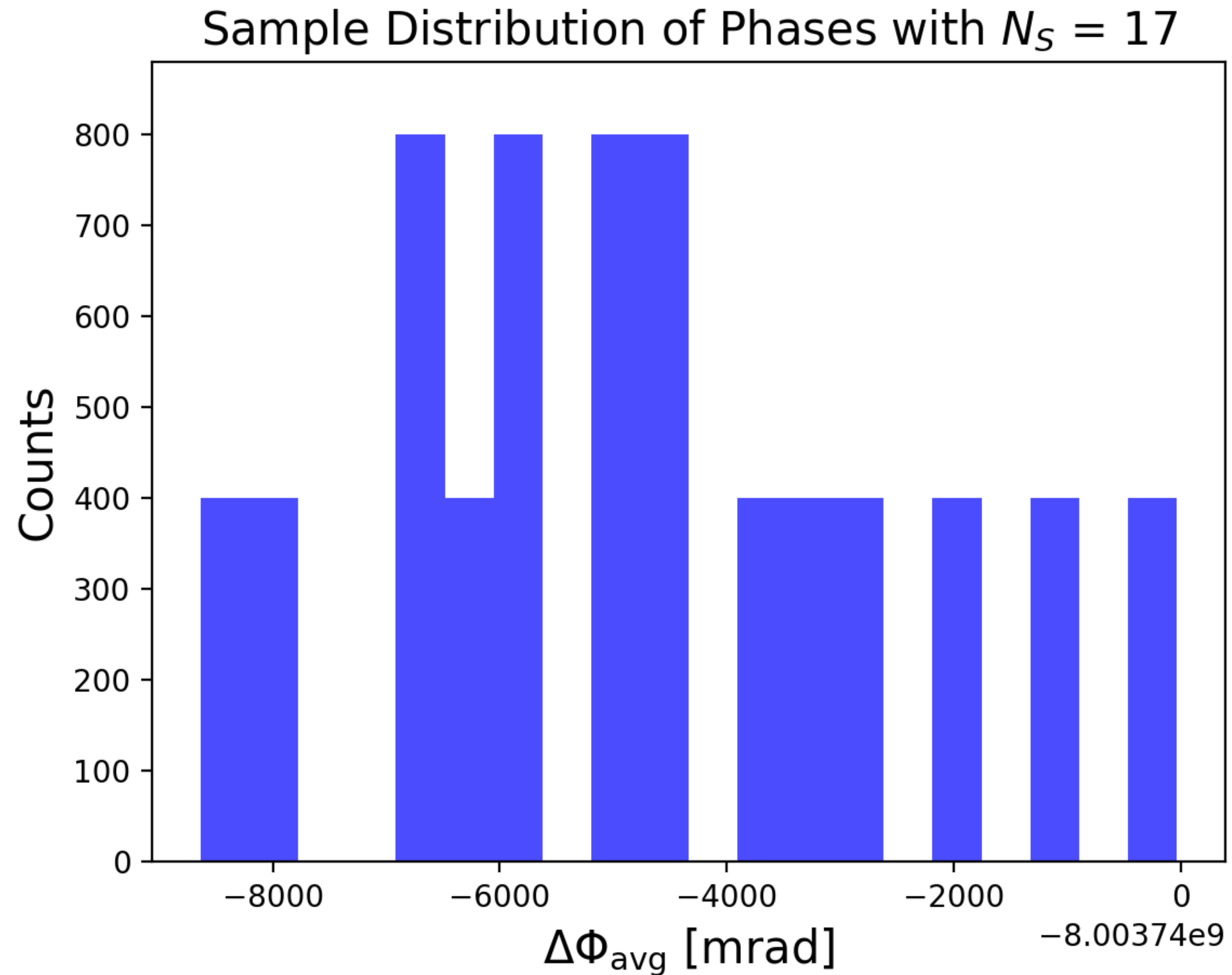
6. Phase sampling

Sample the phase $\Delta\Phi$ of an interferometer with N_I atoms with given spreads in the initial conditions

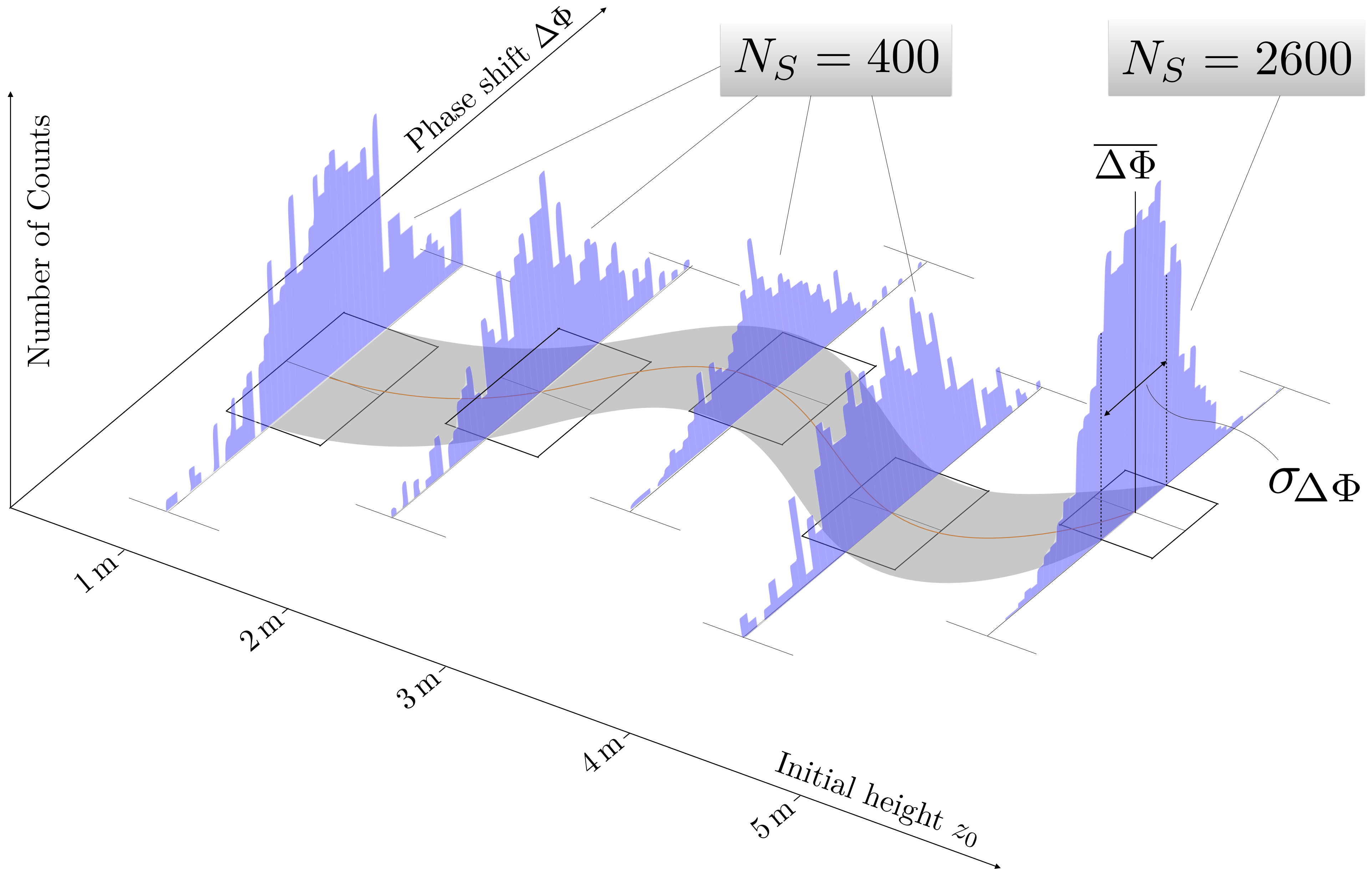
$$\Delta\Phi(\delta z(t), \delta \Phi(t), \delta g(t), \delta \phi(t))$$

Calculate and save the average phase.

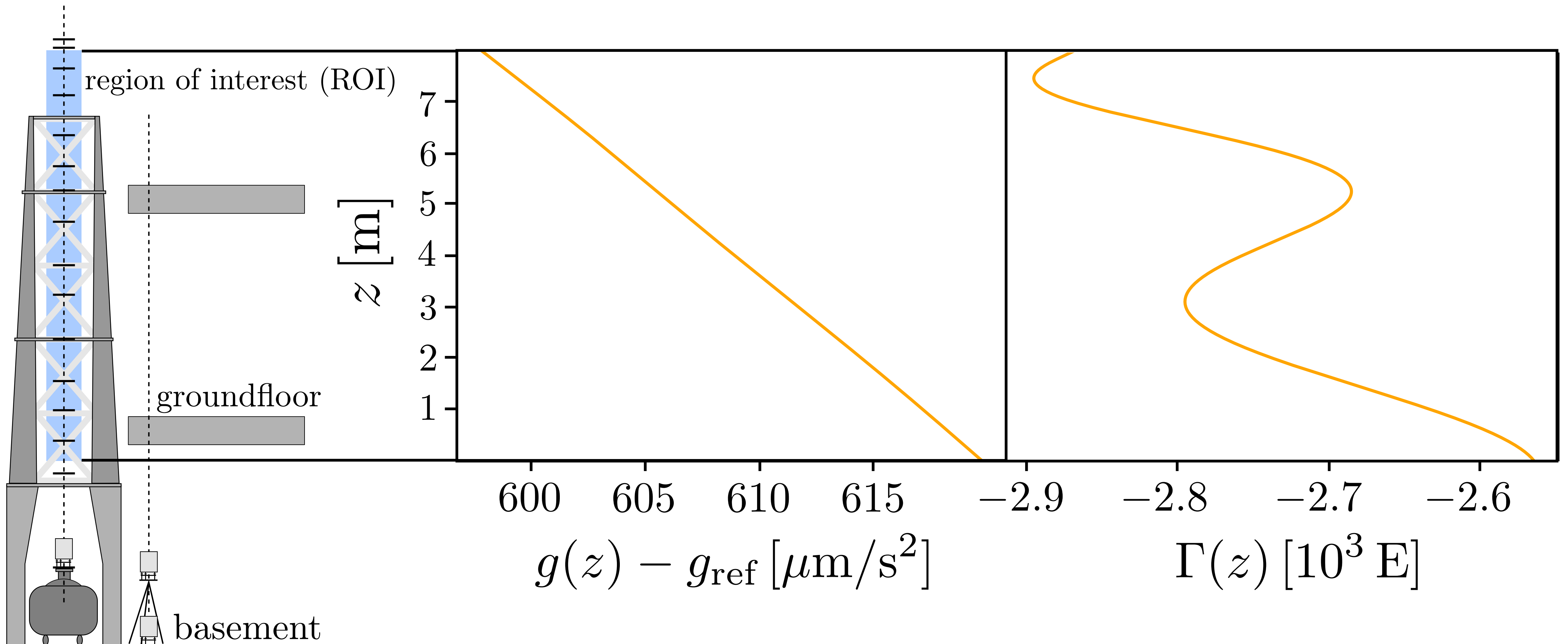
Result: Phase shift statistics



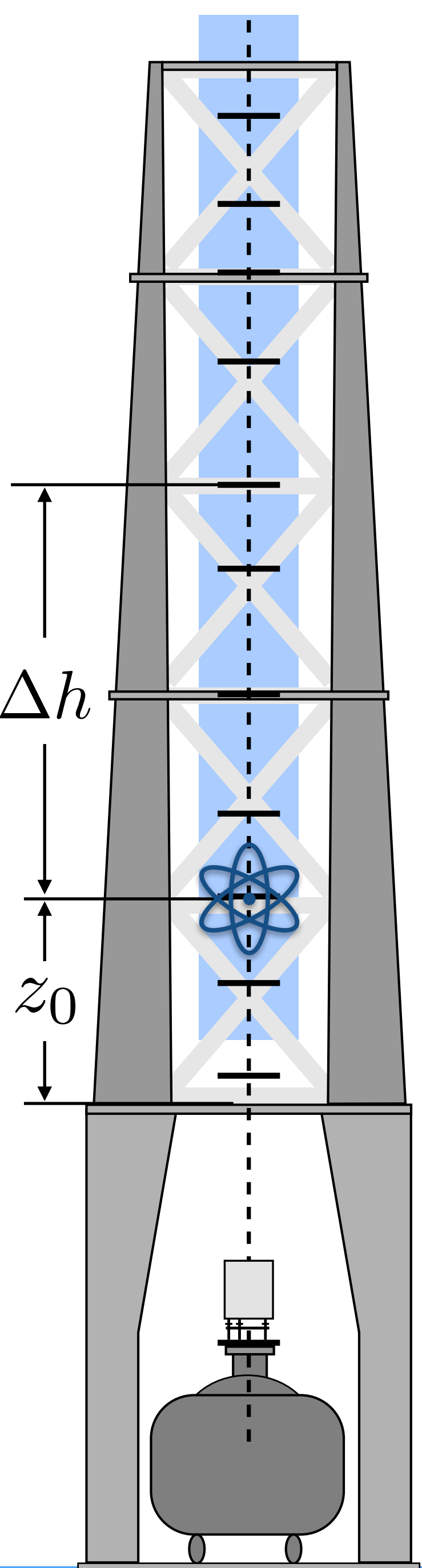
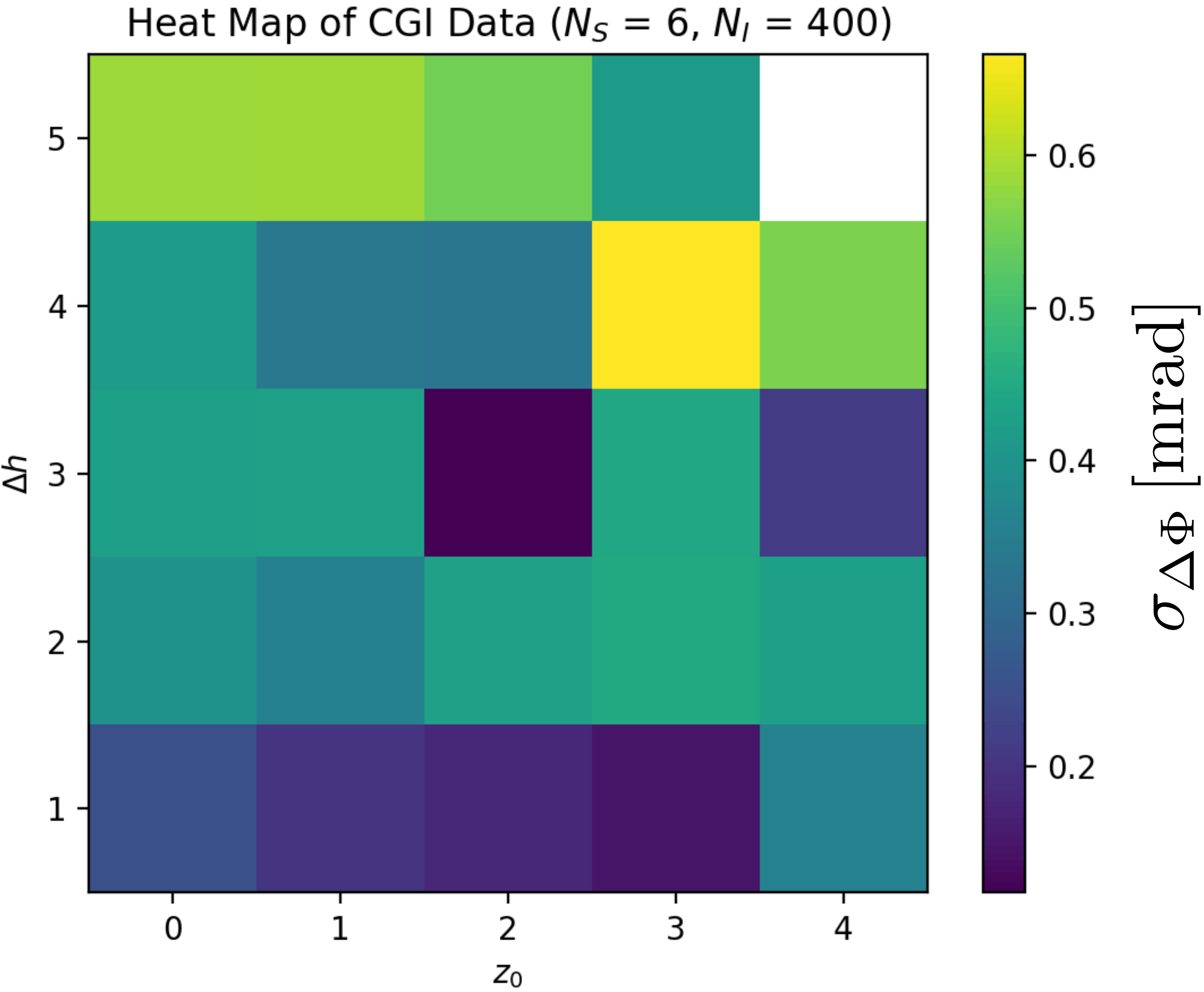
Result: Phase shift statistics



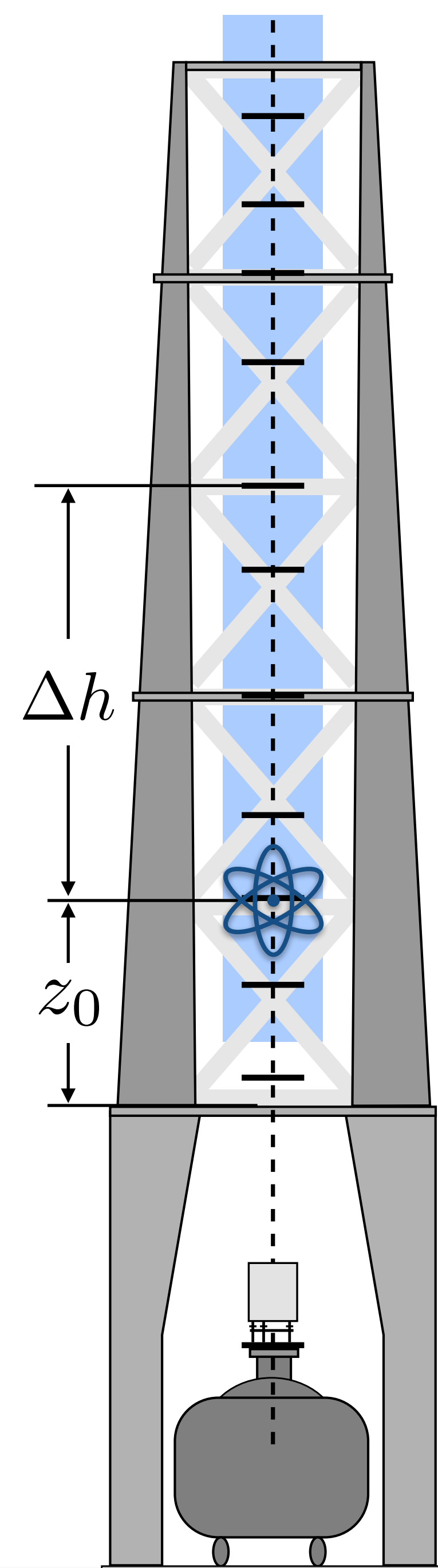
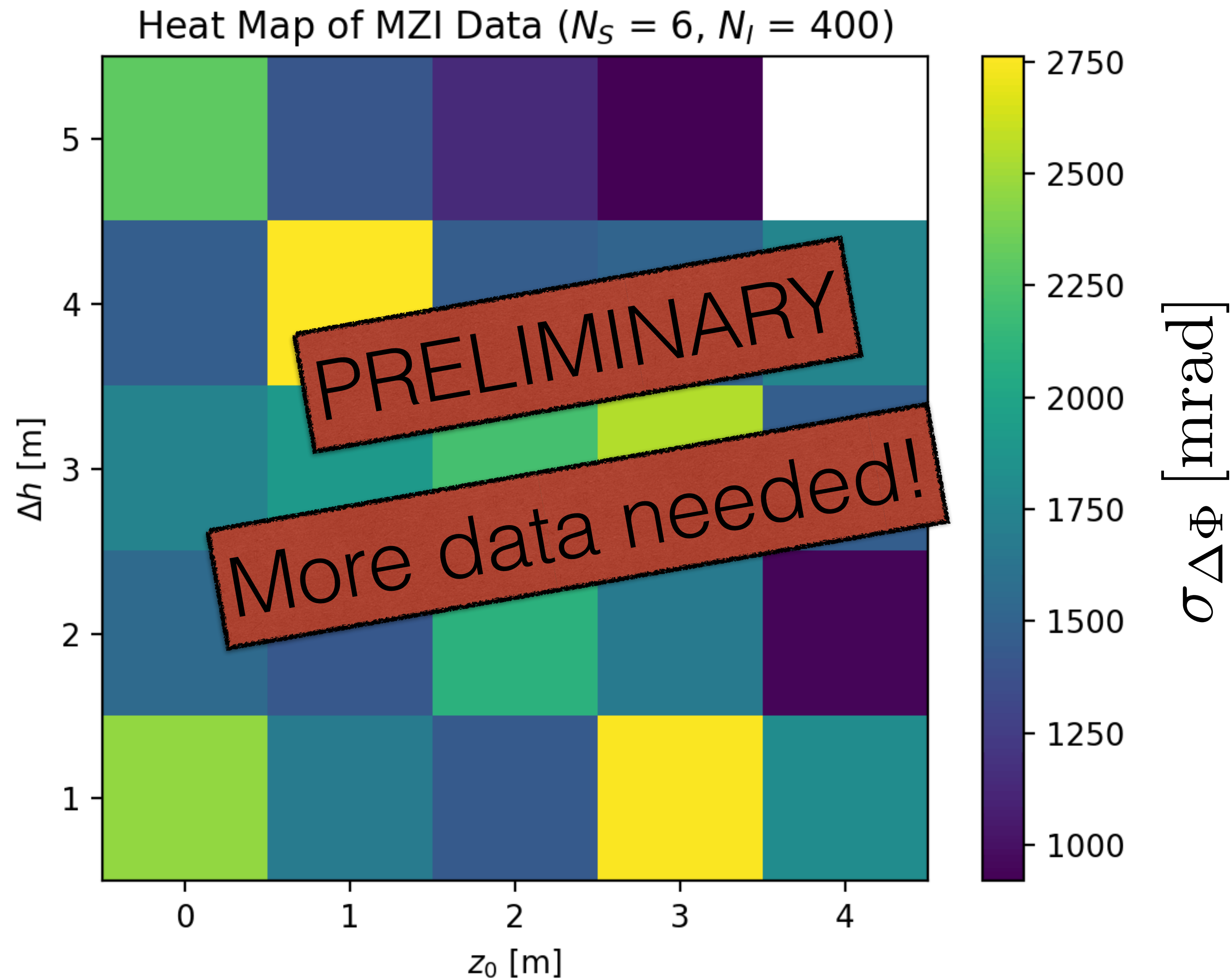
Where shall I operate my interferometer? How high shall I shoot the atoms?



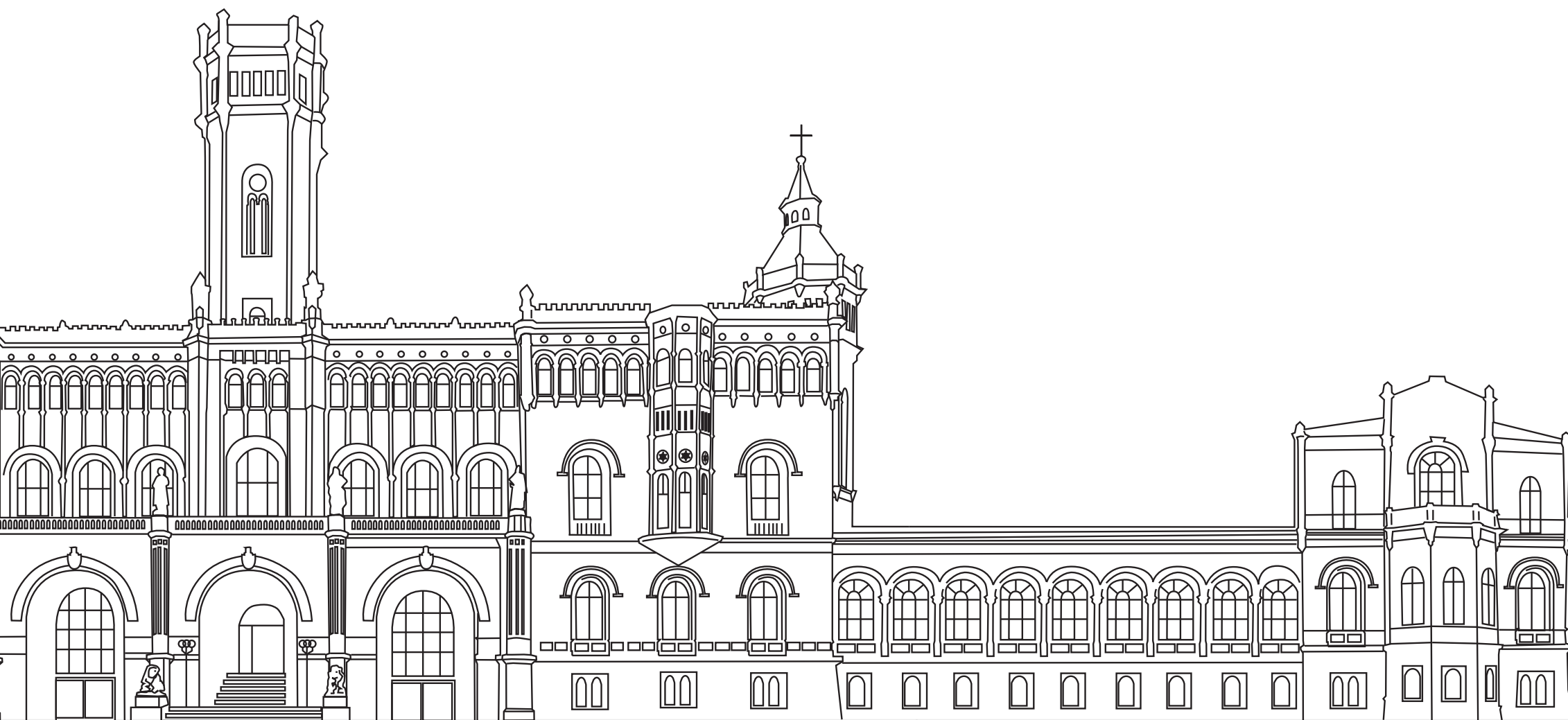
Stability Analysis: CGI



Stability Analysis: MZI



5. The ghost in the machine 👻



The ghost in the machine

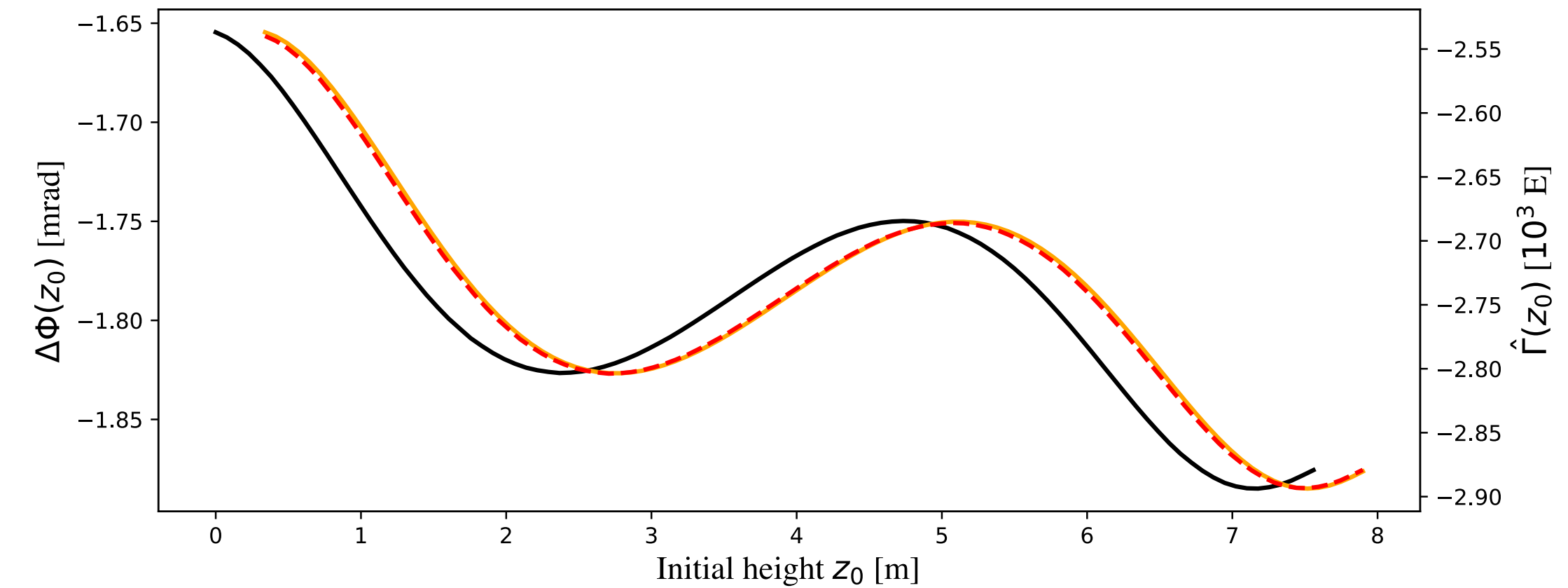
Code

Machine

Result

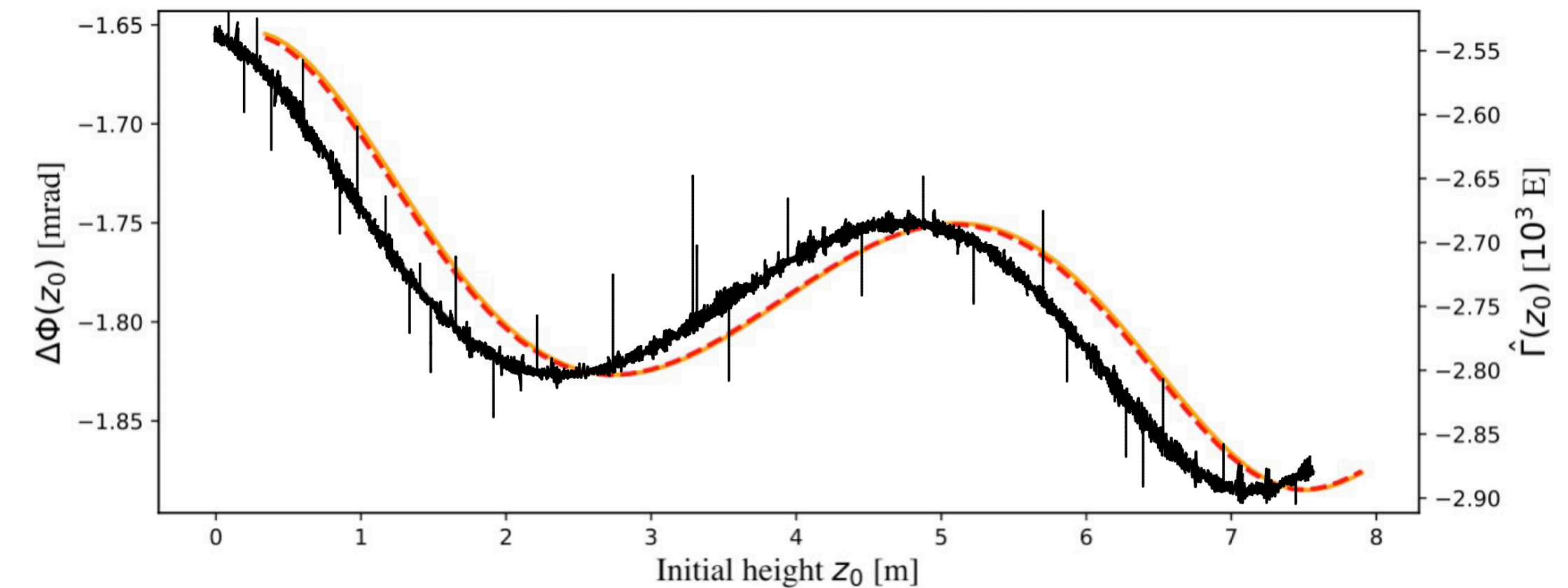
Same Python code.
Same libraries.
Same everything.

MacBook Pro
Intel CPU

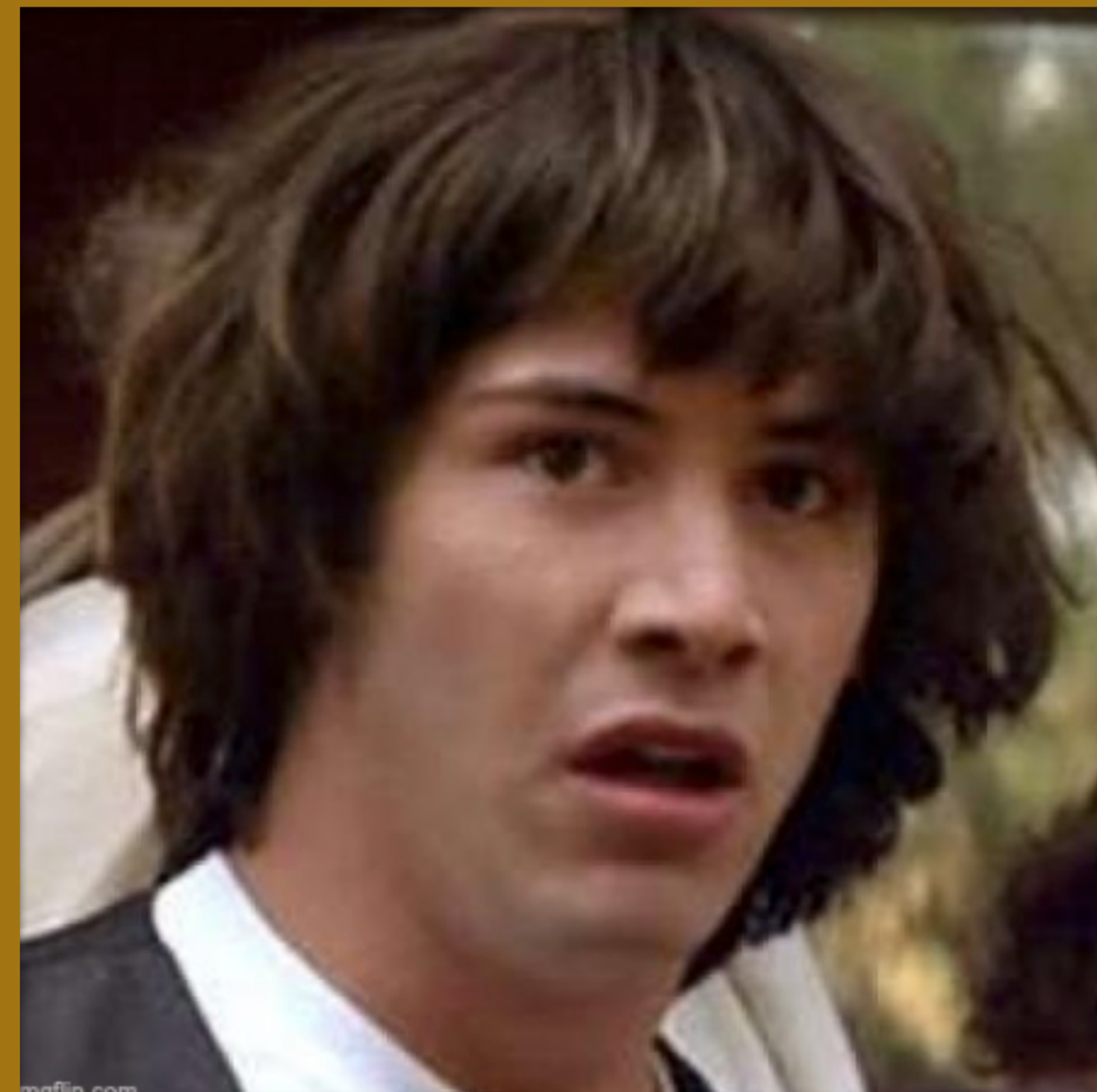


VS

MacBook Pro
Apple Silicon CPU



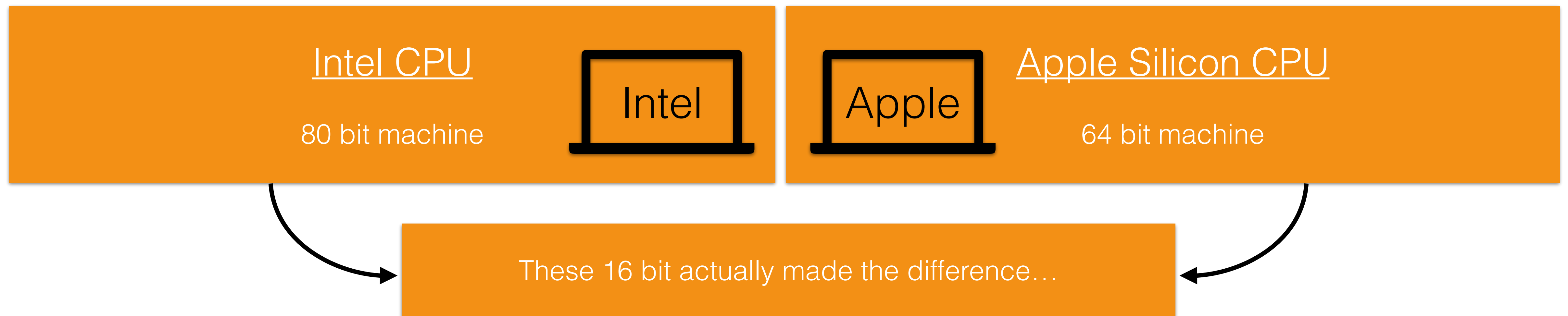
My reaction



*“Delusional state of self”
Acrylic on canvas, 2025.
— Artist unknown.*

The ghost in the machine

- **Double** precision was not enough in my code —> Changes in $g(z)$ in VLBAI are too small for this.
- Python solution: Use **LongDouble**. Because this is equivalent to **Quartic** precision, innit?
- No, it's not. The **LongDouble** class tells the CPU to use the highest accuracy it supports natively.



The ghost in the machine

Solution: We use the Python library “Decimal”.

Idea: Numbers are stored as a string with any desired precision.

Problem: We leave “GPU territory”.

Parallelization now: CPU based using Python library “Dask”.

Future Parallelization: Natively compute quartic precision NVIDIA Blackwell architecture. Quite expensive though...

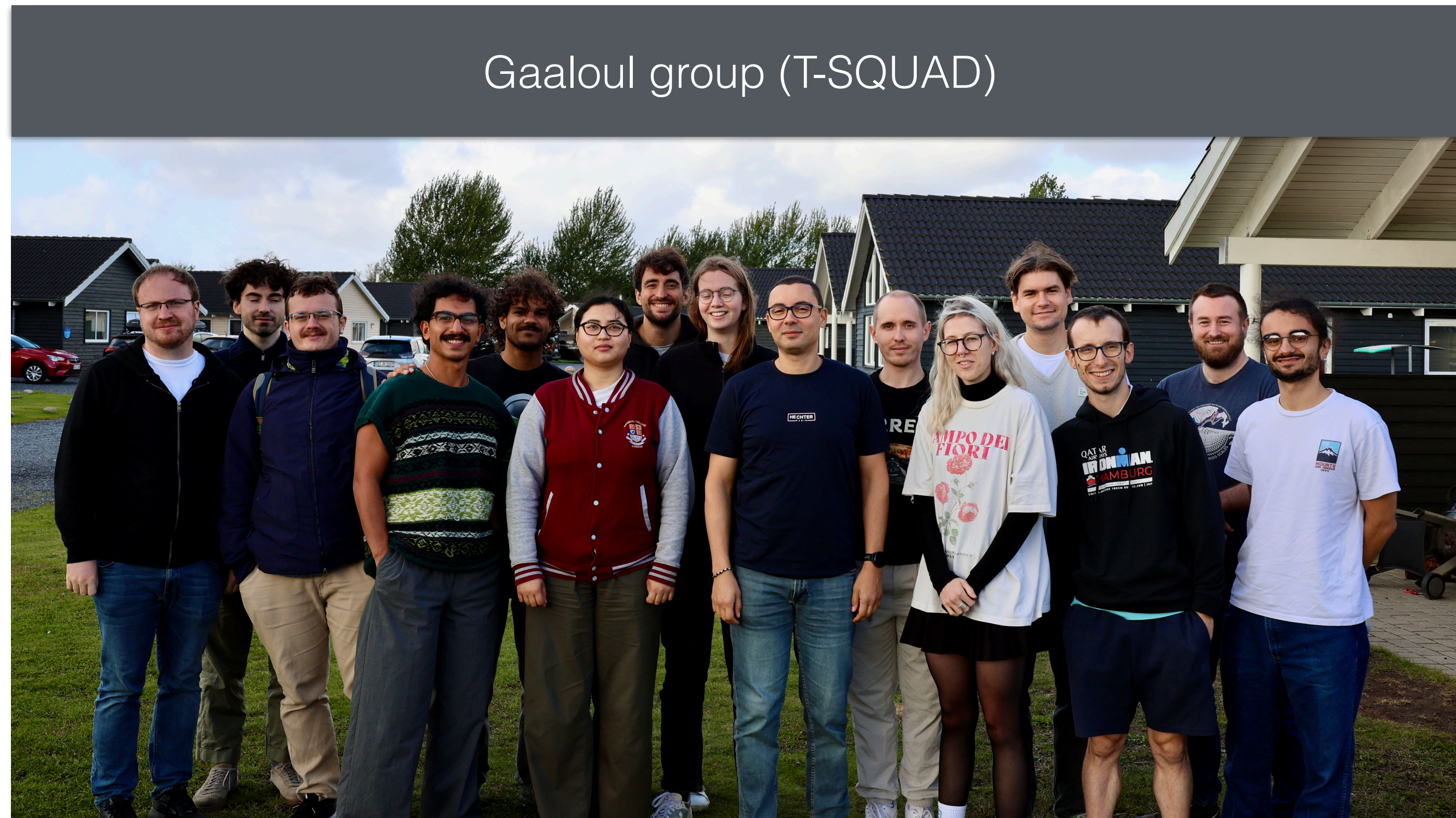
Way faster

Expensive hardware
≠ Open source spirit

Thank you for listening!
Any questions?

And also thanks to those people:

Gaaloul group (T-SQUAD)



We looked @ this code

Quick links:

PPN paper:



Curvature paper:



PPN algorithm:



Curv. algorithm:

